NOTE TO FILE
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Further Study of the shape of the AM curve

1 References

A. Email Boyle to Hall 150104
B. Email Hall to Boyle 150105
C. 150105 Atwood's Machine - Expanded Graphs R1.xlsx
D. 150101 NTF Atwood's Machine R4.docx
E.

2 Purpose

To do some further investigations of the shape of the curve that is characteristic of the output of power of Atwood's Machine.

3 Background

At Ref A I mentioned to Dr Hall that the curve was asymmetric. I was concerned that this might be an error. But he confirmed that it was/is asymmetric. He pointed out that in Odum's "Systems Ecology" book (1983) there was a figure (7-20), with associated mathematics, that showed it to be asymmetric.
Email – Boyle

4 Discussion

At Ref C I put together a variety of combinations of E (Efficiency) and P (Power) where I am examining the shape of the curve. The physical scaling constant is not needed to examine shape, and so has been set to 1. What remains of the equation describing the curve is:

$$P(E) = \sqrt[2]{\frac{E^2(1-E)}{(1+E)^3}}$$	Equ 01

This equation was derived at equation 20 of Ref D. In this note I distinguish between $P_S(E)$, the original AM curve at equation 20 of Ref D, P(E), which is just the variable part, as per equ 01 above, and $P_{index}(E)$, which is a scaled version of P(E) which I introduce below. Most of my graphs are of $P_{index}(E)$, but the other two can be recovered by multiplication by the appropriate scaling constants.

4.1 Wolfram Alpha

I fed this into Wolfram Alpha at Ref E and got a wealth of information.

E.g. The real domain is the interval [-1 < x <= 1]. This means it is defined for negative efficiencies. I doubt it has real meaning for negative efficiencies. This would correspond to an AM in which M_H would be lighter than M_L, and the machine would not function. Another way to say that is, the machine has reached the end of its useful cycle. Or, the machine is primed to work in reverse? As I say, no real meaning. Nevertheless, it has a VERY peculiar shape over the entire domain. (See Figure 01 below.) At x = -1 there is a vertical asymptote. At s=0 it looks like an absolute value function. At x=1 it looks like a collapsing function.

Wolfram alpha also provides a series expansion for each of x=-1, x=0, x=1, and x=∞. Really cool! But I don't see a use for them at this point.

Wolfram alpha also provides a derivative function and a definite integral. The derivative is of interest.

$$\frac{dP}{dE} = \frac{d}{dE}\left[\sqrt[2]{\frac{E^2(1-E)}{(1+E)^3}}\right] = \frac{E-2E^2}{\sqrt[2]{\frac{E^2(1-E)}{(1+E)^3}}(1+E)^4}$$	Equ 02

Setting this equal to zero and solving for E will tell me the point where P(E) is at a maximum value. That's not an easy thing to do analytically.

Thankfully, Wolfram Alpha also provides the answer for that. A local maximum occurs at x = ½ and P(½)=1/(3*root(3))=0.19245. This concurs with Dr Hall's statement that maximum power occurs when x = ½.

Using this information, and, basically, just playing with various graphs as questions arose, I produced seven graphs. I reproduce them here one per page.