NOTE TO FILE:
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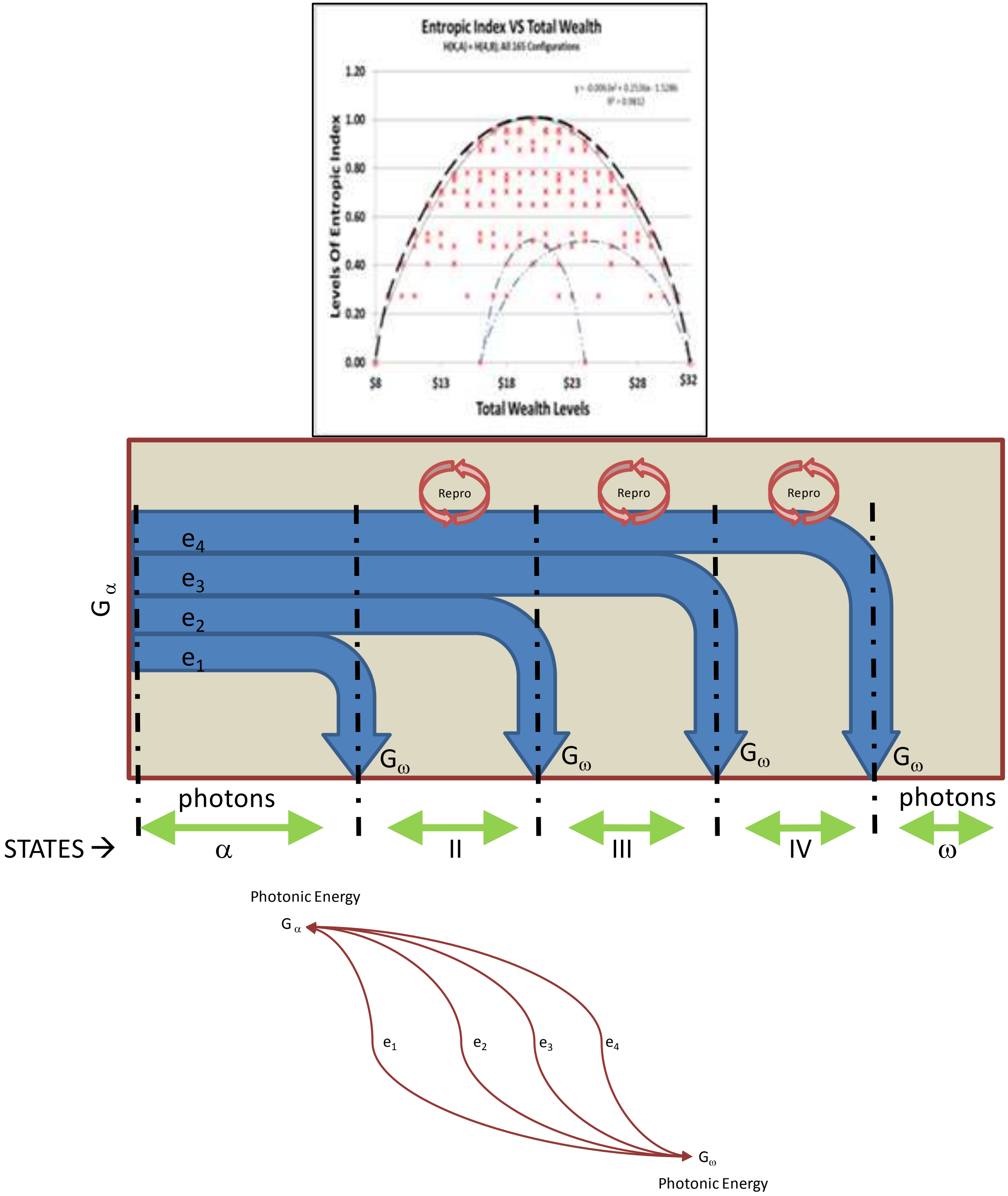
R5: Dated 170330

On Efficiency and Quality Transfer Along Energy Chains

Frontispiece



Contents

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X. Black Body Radiation – (https://en.wikipedia.org/wiki/Black-body_radiation)
Y. Planck’s Law – (https://en.wikipedia.org/wiki/Planck's_law)
Z. Wien’s Displacement Law – (https://en.wikipedia.org/wiki/Wien's_displacement_law)
AA. Sun’s Radius – (https://en.wikipedia.org/wiki/Solar_radius)
BB. Astronomical Unit – (http://www.space.com/17081-how-far-is-earth-from-the-sun.html)

2 - Background

This diary note connects to many different lines of enquiry written up previously.

- Ref A is a paper in which Drăgulescu and Yakovenko describe the BDY model, an agent-based computer model which generates entropy as it functions. I reproduced their models, along with one of my own design called "Model I", in a piece of software called EiLab (Entropic Index Laboratory).
- Ref B is an unpublished paper in which I studied the production of entropy in Model I within EiLab, and defined a concept that I called the Entropic Index as $E_i \equiv S / S_{max}$. In that paper, I also came across a curious fact that in a single-step transition from state α to state β, the size of the change in entropy ΔS was somehow related to the ratio of probabilities $\pi(\alpha\rightarrow\beta)/\pi(\beta\rightarrow\alpha)$. Here, the pi notation is as developed, I understand, in the Ref C document, and is interpreted as "the probability that state α transitions to state β" divided by "the probability that state β transitions to state α".
- Ref C is a paper (of which I could not find a copy, and which I have not yet read) in which Crooks et al develop an argument to the effect that for any two states α and β, the change in entropy between the states is $\Delta S = \ln(\pi(\alpha\rightarrow\beta)/\pi(\beta\rightarrow\alpha))$ where each probability is calculated for all possible transition paths. My result in Ref B was for a one-step single-pathed transition. This was a really cool discovery.
- Ref D is a paper in which Jeremy England uses Crooks' discovery to make deductions about self-replication in general. Self-replication is a key component of persistent systems such as evolve under the effects of the maximum power principle (MPP).
- Ref E is a diary note in which I examine the implications of my insights gained from the study of England's Ref D paper. In this note many ideas came together for me, and many possible avenues of study were opened, of which I have followed up on few.
- Ref F in an unpublished paper in which I first examined the nature of the curve that forms a boundary for an entropy-generating system. As a result of my attention to Refs C, D and E, I later revised it to propose, as a by-product, a definition for the grade (or quality) of an entropy-generating system (in Prigogine's terms, a dissipative structure) using the concept of entropic index. My proposed definition is $G \equiv (1-E_i) = (1 - S/S_{max})$.
- Ref G is a diary note in which I revisit Yakovenko's concept of entropy $S \equiv \ln(A!/\pi(a_i!))$ connect it to the Crooks/England concept of $\Delta S = \ln(\pi(\alpha\rightarrow\beta)/\pi(\beta\rightarrow\alpha))$. The $pi(a_i!)$ in Yakovenko's formula is the discrete multiplication operator (i.e. $a_1! \text{ x } a_2! \text{ x } ... \text{ x } a_n!$), and not the transitional probability indicator.
- Ref H is a paper by A. J. Lotka in which be proposed that persistent energy pathways, such as trophic chains, evolve to function at maximal rates of energy flux, i.e. at maximum power. I believe this means they evolve to capture ever more of the available sources of energy, up to the limits of availability of current technology, and then expand technology to open new sources of energy, as long as suitable sources of energy are available. Here "technology" means either mutation of DNA or engineering discoveries.
- Ref I is a paper in which Odum and Pinkerton proposed their own version of the maximum power principle, inspired by Lotka's writings, but deviating from them slightly, in which they argued that persistent energy transformations evolve to function at maximal rates of energy flux. Two significant differences I see between Odum's MPP and Lotka's MPP are (a) Lotka is talking about chains of transformations whereas Odum is talking about single

transformations; and (b) Lotka is proposing that the degradation of energy will be maximized for a persistent form of a chain, over the length of the chain, whereas Odum is proposing that the degradation of energy will be minimized within a persistent form of transformation. These two proposals are not necessarily incompatible, but I want to come to understand how they function together.

- Ref J is a diary note in which I examine the implications, arising out of my thoughts about Odum's Ref I paper, for energy transformations. In that note I prove (by exhaustive trial and error) that all forms of interaction between Income (I), Costs (C) and Benefits (B) curves (i.e. ICB curve sets) would either evolve to non-persistent transformations in which the flux of benefits is zero, or to persistent transformations in which the flux of benefits is maximized at some intermediate level of efficiency.
- Refs K and L are agent-based computer models in which I attempt to model both energy pathways (Lotka's MPP in OamLab) and energy transformations (Odum's MPP in MppLab) in a single functioning system.
- Ref M is Odum's opus work in which he describes the MPP, and when reading pages 100 to 106 I was driven to try to figure this all out in a way that makes sense. Chapters 1, 7 and 26 have discussions about the MPP. On page 102 there is a section on "chains of energy quality" that led to the writing of this diary note.
- Ref N is a teaching aid produced by Truls Gundersen. It can be downloaded from the internet at www.ivt.ntnu.no/ept/fag/tep4120/innhold/Exergy%20Light%20Version%203.pdf.
- Ref O is a paper in which Yakovenko presents a bare-bones formula for entropy – one that works for very small numbers of agents. E.g. Boltzmann was worried about 10^{23} molecules, whereas my model economies have only a few hundred agents, at most. Model I of EiLab was specifically designed to study the production of entropy in a model economy with conserved capital. I have since implemented entropy calculations using Yakovenko's ideas in my PSoup (evolution demo) and ModEco (economic model) to find that both are spectacularly insightful.
- Ref P is a diary note in which I examine some of the behaviour of Atwood's Machine.
- Ref Q is a diary note in which I examine the application of entropy calculations to agent-based economic and ecological models.
- Ref R is the Ph.D. dissertation in which C.A.S. Hall defined EROI, and Ref S is a recent book in which he expands greatly on its implications for modern global societies.
- Ref T is an interesting paper in which he considers the Earth as a thermodynamic entropy-producing engine. It is the sixth in a series of papers published over several years (1959-1965) in which he applies thermodynamics to a variety of questions relating to evolution and the operations of the biosphere. He refers to the previous papers in this one, and the bibliographic references are very sparse:
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  - Blum, 1951, "Time's Arrow and Evolution".
- Other Refs are to relevant Wikipedia articles.

3 - Purpose

In the writings of H. T. Odum (e.g. Refs I and M) there is a great deal said about energy quality, and it is all deeply intriguing and attractive, but I have not seen much mathematics around the topic that I find totally satisfying. I will not try to write a critique of the math that I have found, as that does not meet my need to find something that is satisfying. I choose to pass over that step here. So, my purpose in this note is to try to come up with something more satisfying that is, nevertheless, consistent with the math, views and arguments of H. T. Odum as I understand them and interpret them. I also find the briefly stated ideas of A. J. Lotka (Ref H) to be very interesting. So, this note is aimed at understanding some aspects of energy quality transformations (Odum's term) as the energy passes through an energy pathway (Lotka's term).

In general, the purpose of this note is to explore the mathematics of chains of energy transfers by using a caricature of a trophic chain as my working example, and using equations mentioned in the background section.

4 - Discussion

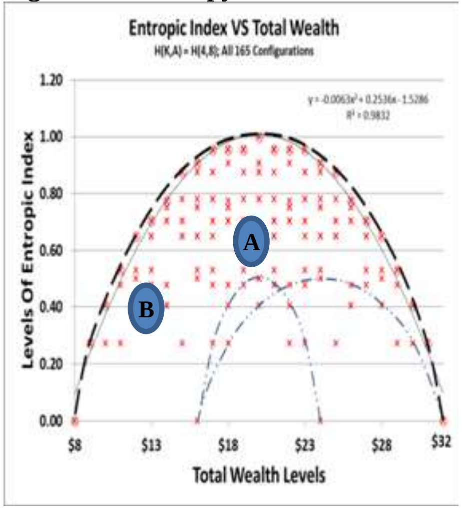
4.1 - On Grades of Energy

In the Ref F diary note I develop a formula to define the grade. It is based on work I did on model economies described in Refs B and F. I was delighted to later discover the teaching aid by Truls Gundersen at Ref N. In that, when describing equations (5) and (6) he talks about exergy as being the useful part of the energy, and presents at formula very similar to mine, which can be converted to mine under the right circumstances. Here's the formula for grade of energy, from Truls' Ref N paper, equation (6).

$W_{max} = Q \cdot \left(1 - \frac{T_0}{T}\right)$	Equ 01

About this equation, Truls says "The Exergy content of heat Q is then the maximum amount of work that can be extracted from this arrangement". So, the factor in brackets is the quality factor. But entropy is defined by Clausius as S = Q/T. In this case, the temperature is changing, and the amount of energy is not, when moving from T_0 to T. If we set $S_0 = Q/T_0$, and $S = Q/T$, then we get $T_0 = Q/S_0$ and $T = Q/S$. According to the explanatory text, in this case T is the initial temperature, and, curiously, T_0 is the temperature of the cold reservoir, at 25°C. This means that S_0 is the higher level of entropy, the maximum amount that can be achieved when the system relaxes to the temperature of the reservoir. Replace S_0 with S_{max}, and substitute into equation 01 to get equation 02.

Figure 01 – Entropy Vs Wealth State



	Equ 02

$W_{max} = Q \cdot \left(1 - \frac{S}{S_{max}}\right)_{constant\ energy}$	

So, using thermodynamic arguments, I come up with the same grade (or quality) factor as I got when studying entropy in a model economy. That is cool, I think! There is one small caveat. This thermodynamic result is limited to constant energy, but my version is more general, allowing for a range of energies (See Figure 01). This means that my definition of S_{max} is more general than Truls' definition of S_{max}. Mine is the peak of the entire curve (Point A), for all levels of wealth, whereas Truls' is where a vertical line meets the curve (e.g. point B).

So, the formula for grade of energy (or of a model economy) is:

$G \equiv \left(1 - \frac{S}{S_{max}}\right)$	Equ 03

I am going to make the rather bold assumption that I can relax the "constant energy" constraint for the energy/thermodynamic case in the same way that I can for the case of model economies.

After some thought, the problem of how to identify what S_{max} is continues to trouble me. One idea that I like has to do with the concept of black body radiation, and the microwave background radiation in deep space. This would apply to S_{max} from a thermodynamic perspective, and not for economic systems. To address that (economic) aspect of the problem, I think I need to go back to the EiLab application and design an economic model that is built for that purpose.

So, for thermodynamics, I would think S_{max} would be the entropy of this system when in equilibrium with the Universe in a state of heat death. At that point, the entropy of the universe would be maximized, and, if entropy of subsystems can be computed, it would also be maximized. THAT is the S_{max} I need. Clearly, the universe is not in a state of heat death at the moment, but I don't think it is unreasonable to assume that the background radiation (often described as the echo of the Big Bang) could stand in for heat death. As I understand it, this background radiation has little or nothing to do with the Big Bang. There is a low level of radiation detectable in all directions. Called background radiation, it has a distribution that follows Plank's law, and is consistent with black body radiation. I would guess that this radiation is, rather, final state radiation, after the second law has extracted all of the 2nd law tax extractable. The question then becomes, does the current state of the background radiation indicate the ultimate final state of heat death? It is pegged, currently, at about 2.7 degrees K, which is pretty low. Over the next trillion years, is it likely to go much lower, or is it pretty well as low as it will go? I would think it will sink a little lower, but very slowly. We are already living in a pretty old universe. So it would not be far wrong to use the current background radiation as a proxy for heat death, and a proxy for a universe at maximum entropy, and fully achieved internal thermodynamic equilibrium.

4.2 - Five Equations for Entropy

Clausius used a definition of entropy based on temperature:

$$S = \frac{Q}{T}$$

Equ 04a

Q represents a quantity of energy, or heat, and T is the temperature in ° Kelvin. I puzzled for a very long time over the units of measure (in terms of dimensional analysis) of entropy. What are Joules per ° K? It turns out, for purists and extremists, the dimensions of temperature are °K which can be reduced to "Joules per number of degrees of freedom for storing energy". For example, energy can be stored in a molecule in rotational energy, vibrational energy, translational kinetic energy, etc. "degree of freedom" is a dimensionless number, like a count, having a role similar to "cycles" in "cycles per second" for which the dimension is T^{-1}. So, the dimensional character of entropy reduces to Joules per Joule, or, a dimensionless quantity. I.e. entropy is a dimensionless ratio, similar to a percentage, or efficiency rating.

Ludwig Boltzmann wrote the equation for entropy where S is the entropy, k is the Boltzmann constant that scales the answer for thermodynamic applications, and Ω is the multiplicity of the system (i.e. the number of states available to the system having the same energy distribution).

$$S = k \ \ln(\Omega)$$

Equ 04b

In the Ref O paper, at equation (3), Yakovenko provides another more basic definition of entropy in this equation:

$$S = \ln\left[\frac{A!}{\prod_{i=1}^{n}(a_i!)}\right]$$

Equ 05

Note that the scaling constant k is missing, and the argument of the ln( ) function is the multinomial coefficient from combinatorics theory. It is e.g. the number of ways A indistinguishable balls can be placed into n boxes, with a_i balls in box i; and $1 \leq i \leq n$, and $A = \sum_{i=1}^{n}(a_i)$. This works wonderfully well for agent-based models, and is totally consistent with other expressions for entropy, such as Shannon's in which $p_i = a_i/A$:

$$S = -\sum_{i=1}^{n} p_i \times \ln(p_i)$$

Equ 06

When A is larger than 170, the two equations are very close to agreement. The Shannon version can be developed from Yakovenko's version via the application of Stirling's approximation (see Ref Q).

However, I have come across a much more interesting equation for the change in entropy. When preparing the Ref B paper I noted that S was associated with an asymmetric ratio of probabilities that I came to call P_{up}/P_{down}. I noted that when considering two states α and β that were a single transition apart, "the probability of transition from the state of lower entropy to the state of higher entropy" divided by "the probability of transition from the state of higher entropy to the state of lower entropy" was always positive.

The paper by England (Ref D) contained an expression for the change in entropy that included this ratio, found as equation (6) in that paper. The expression for ΔS is:

$\Delta S_{(\alpha\to\beta)} = \ln\left[\frac{\pi(\alpha\to\beta)}{\pi(\beta\to\alpha)}\right] = \ln\left[\frac{P_{Up}}{P_{Down}}\right]$	Equ 07

At the Ref G diary note I show that this is consistent with Yakovenko's definition of entropy in Ref O, and I show that Yakovenko's formula is consistent with Shannon's formula, via Stirling's approximation, at Ref Q.

The really remarkable thing about this formula is that England argues that it is associated with the increase of complexity in energy transformations, which makes it of interest in any discussion of the Odum's maximum power principle. I also note that the originator of this formula, Crooks (see Ref C) showed that the formula held when $\pi(\alpha\to\beta)$ included all paths from state α to state β. In my case, I only proved it for the single step transitions in which there was only one path. I can assume it applies for the more distantly separated states, since Crooks' arguments were general mathematical arguments, and not explicitly physical.

4.3 - Three Equations about Efficiency

In the Ref N teaching aid from Truls Gundersen there is an interesting section on formulae for efficiency. Evidently there are a lot of them and they are all slightly different. E.g. there is one group of formulae that he terms "thermodynamic efficiency" in which there is a ratio of actual ability divided by maximum possible ability. In that sense, I suppose that my entropic index $I_S = S/S_{max}$ is a form of thermodynamic efficiency (see Ref B). Since first developing this definition of entropic index, I have been surprised at the number of places such a formula can be used to make sense of formula about entropy-related phenomena. But, as I show above in Figure 01, I need to take care about what S_{max} means.

I have come across two formulae for efficiency in the writings of H.T. Odum and of C.A.S. Hall, the elder statesman among his students. The formula that I herein refer to as Odum's Efficiency is the formula that he used in his paper of 1955 (Ref I) and is the same as that used by Carnot and commonly used by engineers of all stripes since. So, Odum did not invent it, but for my purposes I wish to compare and contrast the formula that I got from Odum's work with the formula that has made Dr Hall famous.

4.3.1 - About Time and Efficiency

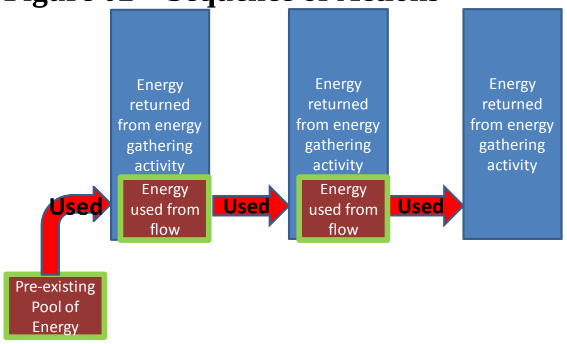
But, before I get into the mathematical details of these three formulae, I need to clarify a question of timing. In all measures of efficiency a ratio is produced two numbers – one being compared with the other. It is not at all clear that time has any role in this, but it is there implicitly. It takes time to transform energy. It takes time to expend it. It takes time to capture or collect it. For example, when talking about the efficiency of a power drill, you might measure the energy consumed over a duration of time, the amount lost to heat, and the amount transferred to the drill bit to do useful drilling. The duration is the same for all three measurements of energy flows (input, waste heat, output) so we tend to ignore them.

However, in most instances of interest to me, the expenditure of energy, or of money, is not constant, and is not simultaneous with the reception of benefits. So, some care needs to be given to how time is handled. I'll use EROI as an example of my concern.

In his Ph.D. thesis (Ref R) and his recent book (Ref S), Dr Hall defines EROI (also called EROEI) as $\frac{Energy\ returned\ from\ an\ energy\ gathering\ activity}{Energy\ used\ to\ get\ that\ energy} = \frac{ER}{EI}$. ER is the "gross energy returned", and not net energy. EI is the energy invested. We might define the "net energy returned" as ER – EI.

In brief, the concept behind EROI is this. If a fish expends energy to catch food, the food must contain at least as much as was expended, or the pre-existing pool of energy in the fish will diminish until the fish is starved. So, on average, the fish's foraging activities must have an average EROI above 1 to survive. There is implied timing in this scenario. The fish has a pre-existing pool of energy stored in the chemical bonds within its body which it invests, and that energy is lost as waste heat as it captures some food. Then, as the food is digested later, there is a 'return' of energy absorbed into the fish's chemical bonds. The time line is: has energy in pre-existing pool→invests energy catching food→invests energy digesting food→has energy returned→replenishes pool of energy.

Figure 02 – Sequence of Actions



In a steady-state fish, each investment of energy must be replaced from the energy returned.

But a fish must capture food many times in its life, so there is an implied sequence of such events, and the activities as bursty with periods of random and quick expenditures (with a Poisson distribution??) followed by long slow periods of digestion overlapping with periods of replenishment. I can make a block diagram of it (see Figure 02).

Of course, the fish must grow, but, for simplicity, consider a steady-state fish. The expenditure of energy related to chasing the food must happen prior to the incorporation of the energy into chemical bonds in the fish. But the expenditure of energy during digestion happens while the energy is being so incorporated. Some of that freshly incorporated energy might replace that used in the chase, and during digestion. So it seems, in a steady-state fish, that the energy stream can be sectioned into three portions: total energy flow (denote it as ER or I, for income), cost of maintenance of the energy flow (denote is as EI or C, for costs), and profits from energy gathering (denote it as ER – EI, or B, for benefits).

I guess my point, that I am making badly, is that the timing of the investment relative to the return would seem to be important. But, if I consider a sufficiently long interval of time, then the important data is the total investment and the total return, in whatever order they happen, and however they happen. There is some value to making the duration of the interval short enough to separate benefit-generating costs from maintenance costs (not always easy), but there is danger in making the interval too short, as it may implicitly exclude some benefit-generating costs that should be included (e.g. costs of lurking, stalking, or failed hunts). So, rather than focusing on a particular action one might look at a longer interval of time and classify a variety of actions (or behaviours) by type of effect, and seek time-averaged measurements.

So the steady-state fish must have an average B >= zero or its pool of energy will be diminished. But, most fish must grow and reproduce to maintain a steady state in an ecosystem because (a) some adults are eaten by predators, and (b) they are pre-programed to die of old age. So, the B has an extra charge against it, beyond food capture and digestions, for growth and reproduction, for avoidance of predators, for hygiene, nest making, homeostasis, etc. When all of these additional charges against B are considered, the EROI of chase/digestion must be much bigger than 1 to pay for them. Chase/digestion is the energy-generating activity. All others are energy-expending activities. Sustainment of the fish's species in an ecosystem requires that the life-long energy generation of the species must address all of these probable costs. I interpret Dr Hall's formula to mean that, if repetitions in time are involved, ALL energy gathering (or energy generating) activities can be divided into the three parts (I, C and B) where I = B + C, and that EROI of the energy-gathering activity = I/C must be greater than one if the energy gathering organism (or organization) is to be persistent.

I have worked with these two formulae for efficiency (i.e. Odum's and Hall's) when studying the energy flows associated with Atwood's Machine (see equation 7 of Ref P), and also when generalizing those ideas to economic flows of benefits (see Ref J). In each I use slightly different notations for variable names. I will present them in both sets of variable names, for comparison purposes.

TABLE 01 – Equations for Efficiency.		
Efficiency in Atwood's Machine	**Efficiency in ICB Curve Sets**	**Equ #**
Definition of Terms		
E_T = total energy E_U = still useful energy E_W = wasted energy	I = Total income B = Benefits (or profits) C = Costs	
$E_T = E_U + E_W$	$I = B + C$	Equ 08
Odum's Efficiency		

$\eta \equiv \frac{Still\ Useful\ Energy}{Total\ Energy} = \frac{E_U}{E_T}$	$\eta \equiv \frac{Profits}{Income} = \frac{B}{I}$	Equ 09

Hall’s Efficiency		
$EROI \equiv \frac{Total\ Energy}{Wasted\ Energy} = \frac{E_T}{E_W}$	$EROI \equiv \frac{Income}{Costs} = \frac{I}{C}$	Equ 10

Isolating I in each, and equating them, I get these equations relating them, in both sets of symbols:

Relation Between Them				
$\eta = \frac{E_U}{E_W \times EROI}$	$EROI = \frac{E_U}{E_W \times \eta}$	$\eta = \frac{B}{C \times EROI}$	$EROI = \frac{B}{C \times \eta}$	Equ 11

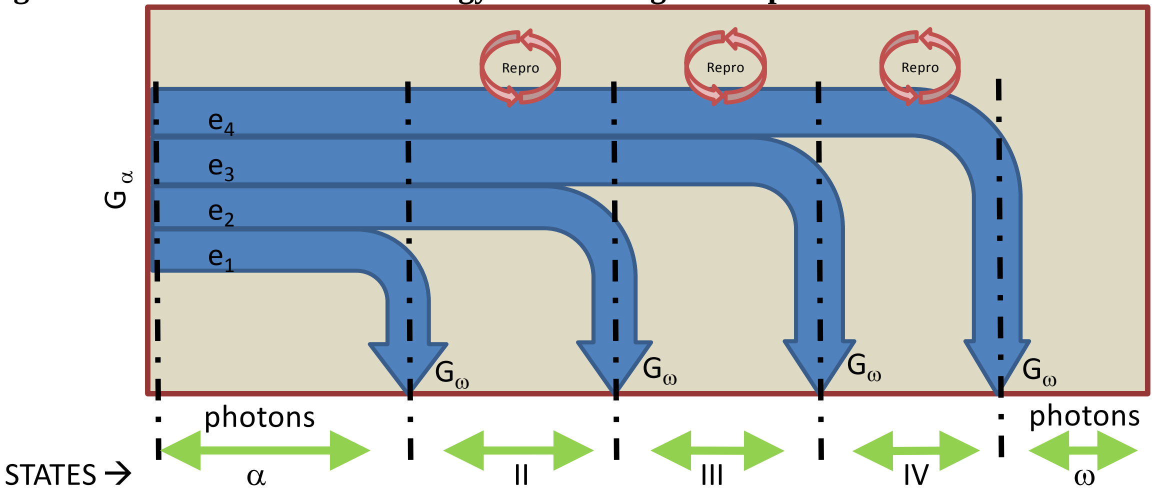
With a little thought, it is clear that:

- by definition (i.e. Equ 09), η can never be less than zero or greater than 1;
- by definition (i.e. Equ 11), EROI can never be less than zero, but has no upper limit;
- the 2nd law of thermodynamics requires that η be less than one for every energy transformation whether considered over a short or long duration; and
- persistence requires that EROI be greater than one for some reasonably long duration, and the qualification of “reasonably long” is highly dependent on circumstances.

4.4 - A Caricature of an energy flow

Consider the caricature of a trophic chain shown in Figure 03.

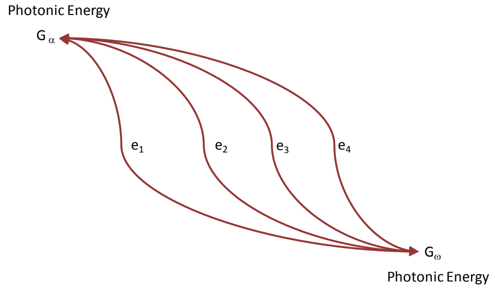
Figure 03 – Caricature of the Energy Flow through a Trophic Chain.



This could be a caricature of any chain of energy transformations, such as:

- extraction, refinement, delivery, and consumption of petrochemical or hydro-electric energy;

Figure 04 – Skeleton View of Fig 03



Energy is conserved, so all energy bands continue into the past and future forever. Fig. 02 is a splice out of the middle of each.

- flow of matter/energy through an organism from consumption to excretion/radiation;
- flow of energy through a cell;
- possibly, flow of capital value through an economy.

Normally this would be diagramed as a series of energy stores, with energy transfers shown between them. I have chosen to focus on the energy streams, leaving out the stores, which are there implicitly. Each "state" is essentially a form of energy store.

My focus will be on an interpretation as a trophic chain, from sunlight to plant to apex predator and to deep space again. I have identified, without intention of loss of generality, five states (five stores), labeled the alpha state (α), the omega state (ω) and three intermediate states labeled using Roman numerals (II, III and IV). The cyclic flow of energy above the three biological states indicates the feedback loop (in propagules?) that maintains the biomass. I am assuming, at this point, that the biomass remains stable, so reproduction does not remove energy from the stream. **This is a significant stationary state assumption that needs to be revisited and changed in a deeper consideration.**

Time is meant to increase, non-monotonically, from left to right. There are four "energy bands" labelled e_i, each tracking the quality of the energy in that band. For simplicity, the grade or quality of each band is either good (high or able to do "useful work") or bad (low or unable to do any more "useful work"). I realize that it can be anywhere in between, but I hope to examine such in between conditions of quality by considering how the bands degrade one at a time, producing several intermediate conditions of degrading quality of energy in the system as we move from left to right. I do not mean to imply that the energy band ceases to exist when the arrow terminates. There should be a continuation of the four bands at the bottom, but the diagram gets too complicated. A skeletonized version of this is in Figure 04.

I show five changes of state (or energy transformations or transitions), indicated by the black vertical dotted lines. The states, and the changes of state, can be described as follows, from the perspective of a trophic chain:

- **Transition ($\rightarrow\alpha$)** – This transition (first vertical dotted line on the left) includes all of the energy transformations from the Big Bang up to the incidence of light coming from the Sun and being captured by the Earth's biosphere via photosynthesis. Photons coming from the Sun are created by a change of state within the Sun, not shown, and the pre-alpha state is not shown, but the diagram could, possibly, be extended to pre-solar states, pre-galactic states, and ultimately the big bang state (of which I have no confidence of it actually having happened, but that is a different question). The immense duration of this interval of time is compressed into the vertical line.
- **Alpha state (fully useful pre-capture photons)** – This is the state of photons of light coming from the Sun. For my purposes, I consider all energy as coming from the Sun in sunlight. I am ignoring energy from nuclear decay, tectonic movement, cooling of the core, tidal movements, etc. to simplify the discussion. So, all of the four indicated fully useful bands of energy are considered to be coming from the Sun as sunlight. Also for the sake of simplicity, I do not distinguish between different kinds of photons, having different energy levels (colours and wavelengths), other than to specify that these are the photons able to participate in photosynthesis, by reason of energy level and circumstance. Those photons of

sunlight that are not captured by plants via photosynthesis are not included, due to my focus on a trophic chain. The Wikipedia articles at Refs U and V have a great discussion of the complexities of photon capture and the efficiencies thereof. I concern myself only with the energy which will effectively be captured and incorporated into plant matter.

- **Transition ($\alpha \rightarrow$II)** – This transition requires time to be completed, and starts about the point where the e_1 energy arrow starts to turn down, and ends when arrow depicting the e_1 energy band comes to an end. Over the duration of this transition, the energy band e_1 transitions from "fully useful" (grade $\cong$ 1) to "totally useless" (grade $\cong$ 0). Grades are explained below. Of course, as described above, and as seen in Fig. 04, the energy of the e_1 band continues to exist (if we can say that energy "exists") and continues forever, in a fully degraded state as "waste heat", while the remaining bands have been converted to chemical potential within the plant. This ($\alpha \rightarrow$II) transition can be considered the transition from energy in photons to energy in plant matter. The means of the transformation is photosynthesis and incorporation into the chemical bonds of the plant matter.
- **State II (plants)** – The still useful energy in bands e_2, e_3 and e_4 is contained as chemical potential energy initially within the molecular bonds of the sugars, and then later in the starches, cellulose and other matter within a plant. The e_2, e_3 and e_4 bands are converted to chemical potential energy in such plant matter, and the e_1 band was degraded during photosynthesis and energy conversions.
- **Transition (II$\rightarrow$III) –** This transition requires time to complete, being the process of consumption of the plant by a herbivore, digestion, and incorporation of the e_3 and e_4 energy bands into the proteins, fats, and other animal matter of the animal. In the same spirit of simplification used in physics problems when friction is ignored, I subsume the "2nd law" loss of energy of the herbivore as it forages and grows into the degradation of energy here depicted in the e_2 band. So, this e_2 band includes all of the energy expended in maintenance activities and growth to maturity, as well as the degradation of the energy in the e_2 band due to inefficiencies in digestion. The e_1 band, of course, was already removed and is unavailable to the herbivore.
- **State III (herbivores) –** The still useful energy in bands e_3 and e_4 is contained in the chemical potential energy within the molecular bonds of the fats and proteins in the herbivore's body. The energy bands e_1 and e_2 are not included, as they are no longer useful, and have been dissipated as waste heat.
- **Transition (III$\rightarrow$IV) –** This transition requires time to complete, being the process of consumption of the herbivore by a carnivore, digestion, and incorporation of the energy band e_4 into the chemical potential bonds within the body of the carnivore. The degradation of the e_3 band includes all of the maintenance and growth activities of the carnivore.
- **State IV (carnivores) –** The still useful energy in band e_4 is contained in the chemical potential energy within the molecular bonds of the fats and proteins in the carnivore's body. The e_1, e_2 and e_3 bands are no longer useful, and have been dissipated as waste heat.
- **Transition (IV$\rightarrow \omega$) –** This transition requires time to complete, and is a kind of roll-up of all processes that bring the continuing e_4 band of energy to a state of totally "no-longer-useful". This involves the growth and maintenance of apex predators, natural death, parasites, detrivores and decomposers. I could complicate Fig. 03 by adding an arrow for each of plants, herbivores, and carnivores feeding into detrivores and decomposers, but that, I think, is a needless complication. The diagram, as it stands in Figs. 03 and 04 is a suitable caricature of the overall energy flow. All of the four bands of energy have been transformed

into "waste heat" in the environment, and transformed into low-energy high-entropy infra-red photons, and dissipated into deep space as background radiation. All of these various transformations to omega state are considered to have happened in the (IV→ω) transition.

- **Omega state (no longer useful photons)** – All four bands of energy that were captured via photosynthesis are no longer useful, and exist as background radiation in deep space. The duration of time from the beginning of the (α→II) transition to the end of the (IV→ω) will vary greatly depending on the path taken for each portion of energy. But I assume that eventually all of the captured energy is degraded and released. That energy now occupies a sphere in space centred on the Earth, and exists as black body radiation. At Ref T Kirkaldy says that Blum (1951) described the radiation leaving the Earth as distributed with a maximum wavelength at 10,000 Å (Ångström). Again, Kirkaldy mentions an "effective temperature", but does not say what it is estimated to be, or how to calculate it.

Before continuing, I need to make one further comment on the (α→II) transition. Some people see this as a concentration of energy, and I suppose that perspective has its value, but I think it is more properly viewed as a dispersion of some portion of the energy in space, offset by some concentration. As stated in the Wikipedia articles (Refs U and V) it requires about 6 photons of light to construct one molecule of sugar in photosynthesis. The transformation is not instantaneous, so the duration of time for this to happen is finite. The molecule of chlorophyll has an effective cross-section, the photons have a speed, and so the volume of space occupied by the parallel paths of flight of the photons prior to absorption can be calculated as the volume of a cylinder, with an associated energy density, and an associated energy grade. After photosynthesis, a portion of that high-grade energy is now concentrated in the relatively small volume occupied by the chemical bonds within the molecule. However, another portion has been degraded to waste heat, and has been dissipated in non-parallel rays, at the speed of light, in a spherical volume centred on the Earth. I cannot estimate the number of infra-red photons released, or how far such infra-red photons can travel before being absorbed by the air, but the volume of space over which this residue of degraded energy is spread is substantially larger than the volume occupied by the sugar molecule. The question in my mind is, how does this compare to the original cylindrical volume, and is the energy now spatially more concentrated or less concentrated? I suspect that it is dramatically less concentrated. I also suspect that if I could construct a formula for entropy of spatial concentration, that measure of entropy would rise as the concentration of energy falls. If we defined a system such that all of this happened within the confines of the system, the thermodynamic entropy of the system would rise (of course, per the 2nd law of thermodynamics), the grade of the energy within the system would fall, and the spatial entropy of the system would rise.

All of that is to say, while I understand that some portion of the incoming energy has been captured and confined to a small space, I am uncomfortable with the language that describes it as concentrated energy, or as higher quality energy. The totality of the incoming energy is very probably less concentrated, and has less quality. We need different more precisely qualified language to talk about it. **Hmmm!**

4.5 - Mathematics Describing Energetics of a Trophic Chain

So, the goal is to apply the above equations to the caricature of a trophic chain as shown in Figure 02. There are some basic mathematical relations. Define E as the sum of the e_i.

$E \equiv \sum_{i=1}^{4} (e_i)$	Equ 12

To find potential simplicities, I also carry along a possible assumption. Define ‘e’ as E/4, and set all e_i equal to e.

$e \equiv \frac{E}{4} = e_1 = e_2 = e_3 = e_4$ and $E = 4e$	Equ 13

4.5.1 - Odum’s Efficiency

I refer to this as “Odum’s Efficiency” here because it is the formula used in his paper of 1955 (Ref I), but it is actually a common definition of efficiency. There are four energy transitions for which I can calculate the efficiency using “Odum’s Efficiency”. The subscripts indicate the transition from state to state.

TABLE 02 – Calculations of η for Figure 03.

Transition	General Efficiency	Equal Sized Bands	Equ #
$(\alpha \rightarrow II)$	$\eta_{(\alpha \rightarrow II)} = \frac{e_2 + e_3 + e_4}{e_1 + e_2 + e_3 + e_4}$	$\eta_{(\alpha \rightarrow II)} = \frac{3e}{4e} = \frac{3}{4}$	Equ 14
$(II \rightarrow III)$	$\eta_{(II \rightarrow III)} = \frac{e_3 + e_4}{e_2 + e_3 + e_4}$	$\eta_{(II \rightarrow III)} = \frac{2e}{3e} = \frac{2}{3}$	Equ 15
$(III \rightarrow IV)$	$\eta_{(III \rightarrow IV)} = \frac{e_4}{e_3 + e_4}$	$\eta_{(III \rightarrow IV)} = \frac{e}{2e} = \frac{1}{2}$	Equ 16
$(IV \rightarrow \omega)$	$\eta_{(IV \rightarrow \omega)} = \frac{0}{e_4}$	$\eta_{(IV \rightarrow \omega)} = \frac{0}{e} = 0$	Equ 17

That is all obvious. When several links of the chain are viewed as a single segment, they multiply.

$(\alpha \rightarrow IV)$	$\eta_{(\alpha \rightarrow IV)} = \frac{e_4}{e_1 + e_2 + e_3 + e_4}$	$\eta_{(IV \rightarrow \omega)} = \frac{1e}{4e} = \frac{1}{4}$	Equ 18

This leads to an equation for a generalized segment of a chain of m transformations T_i, each having efficiency η_i, and being between contiguous parts of a longer chain. For example, each of the following would be well-formed chains: α→II, α→III, α→IV, II→III, II→IV, II→ω, III→IV, III→ω, IV→ω, or α→ω. I want to generalize my notation for this. Denote the initial energy store at the head of the chain segment as either α or β. And denote the final store of the chain segment as either ψ or ω. Both α and ω are reserved for the photonic storages, while β and ψ can represent any storage, with the restriction that if both are used, then β represents the higher entropy storage, and energy moves from β to ψ.

$\eta_{(\beta \to \psi)} = \prod_{i=1}^{m} (\eta_i)$	Equ 19

4.5.2 - Hall's Efficiency (EROI)

Returning to the notation of Figure 03, I can produce a similar table for Hall's "energy returned on energy invested" (or EROI):

TABLE 03 – Calculations of EROI for Figure 03 – Method 1.			
Transition	**General Efficiency**	**Equal Sized Bands**	**Equ #**
$(\alpha \to II)$	$R_{(\alpha \to II)} = \frac{e_1 + e_2 + e_3 + e_4}{e_1}$	$R_{(\alpha \to II)} = \frac{4e}{e} = 4$	Equ 20
$(II \to III)$	$R_{(II \to III)} = \frac{e_2 + e_3 + e_4}{e_2}$	$R_{(II \to III)} = \frac{3e}{e} = 3$	Equ 21
$(III \to IV)$	$R_{(III \to IV)} = \frac{e_3 + e_4}{e_3}$	$R_{(III \to IV)} = \frac{2e}{e} = 2$	Equ 22
$(IV \to \omega)$	$R_{(IV \to \omega)} = \frac{e_4}{e_4}$	$R_{(IV \to \omega)} = \frac{e}{e} = 1$	Equ 23

The interpretation of Figure 03 and the translation into formulae is not so straightforward in this case, and I have had to look at it several times. There are at least two ways to construct this table, and I am unsure which is "correct":

- TABLE 03 – Based on my logic around Figure 02, the "energy invested" must be included in the "energy returned", regardless of timing. For example, the e_1 band is part of the incoming E, and so should be counted top and bottom. This leads to the presentation in Table 02.
- TABLE 04 – However, I am bothered by the fact that the last entry in the table has EROI of 1, when there is, in fact, no return on investment, and the last smidgen of energy is being degraded by decomposers and detrivores. So, TABLE 04 is the alternate interpretation.

TABLE 04 – Calculations of EROI for Figure 03 – Method 2.			
Transition	**General Efficiency**	**Equal Sized Bands**	**Equ #**
$(\alpha \to II)$	$R_{(\alpha \to II)} = \frac{e_2 + e_3 + e_4}{e_1}$	$R_{(\alpha \to II)} = \frac{3e}{e} = 3$	Equ 24
$(II \to III)$	$R_{(II \to III)} = \frac{e_3 + e_4}{e_2}$	$R_{(II \to III)} = \frac{2e}{e} = 2$	Equ 25
$(III \to IV)$	$R_{(III \to IV)} = \frac{e_4}{e_3}$	$R_{(III \to IV)} = \frac{e}{e} = 1$	Equ 26
$(IV \to \omega)$	$R_{(IV \to \omega)} = \frac{0}{e_4}$	$R_{(IV \to \omega)} = \frac{0}{e} = 0$	Equ 27

The difference between tables 3 and 4 seems to revolve around the question of whether the "energy returned" is the "net energy returned" or the "gross energy returned".

I suppose that is the issue I was trying to address when I drafted Figure 02.

When several links of the chain are viewed as a single segment, however, they do not multiply, and a simple formula seems to be difficult. I have given this considerable thought when reviewing the books of Dr Hall, and I had no answer to this riddle at that time. However, it seems to me that the logic of equation 13 above applies equally to a chain of energy transformations as it does to a single transformation. So, combining equation 13 with equation 21 I get this:

$$EROI_{(segment)} = \eta_{(segment)} \frac{E_T}{E_C} = \left(\prod\nolimits_{i=1}^{k} (\eta_i)\right) \frac{E_T}{E_C}$$

Equ 26

Where:

- E_T is the total energy flowing into the head of this segment of the chain;
- E_C is the total energy degraded by all transformations in this segment; and
- η_i is the Odum efficiency of each transformation in the segment.

This equation might provide an easy way to calculate the EROI at the refinery, at the gas pump, or at the doorstep when being picked up by a taxi cab. Hmmm? Or maybe not.

4.5.3 - Entropy Production and Grade of Energy

This gets tricky, and I am working at the very edge of my understanding of these things, so this becomes quite speculative.

In Odum’s Ref M book (page 117) he mentions energy being stored in photons. Stages α and ω are meant to represent such storage and transport of energy within a flux of photons.

Denote the grade of the energy in the α state as G_α, denote the entropy associated with the photons in the α state as S_α, and do similarly for the ω state. (See Fig. 03.) I need to make some basic assumptions about the relationships between quality of energy (i.e. grade of energy) and the associated entropy. I think if I focus, at first, on the first and last states, it will be easier:

- I suppose the grade of energy in the photons in the α state would be close to 1. $G_\alpha \cong 1$.
- I suppose the grade of energy in the photons in the ω state would be closer to 0. $G_\omega \cong 0$.
- The grade of the ω state would be less than the grade of the α state. $0 < G_\omega < G_\alpha < 1$.
- The entropy associated with the ω state would be more than the entropy associated with the α state. $0 < S_\alpha < S_\omega < S_{max}$.
- I need to assume, I think, that entropy can be localized or partitioned, and that the total entropy in a system can be viewed as the sum of the parts of entropy found in parts of the system. So, if ${}_{e_i}S_\alpha$ is the entropy in energy band e_i when the system is in state α, then $S_\alpha = \sum_{i=1}^{4}\left({}_{e_i}S_\alpha\right)$. This additive quality of entropy is a natural consequence of the logarithm in the formula, as evidenced by Shannon’s equation (see equation 06).

From equation 03 we see that $G_\alpha \cong 1$ implies that $S_\alpha \cong 0$. The energy in a photon can be calculated using the formula E = hc / λ, where h is the Planck constant, c is the speed of light in vacuum, and λ is the wavelength of the photon. We can therefore refer to the α-state photons as high-energy low-entropy short-wavelength photons. Similarly, we can refer to the ω-state photons as low-energy high-entropy long-wavelength photons.

$$G_\alpha \equiv \left(1 - \frac{S_\alpha}{S_{max}}\right)$$

Equ 27

$$G_\omega \equiv \left(1 - \frac{S_\omega}{S_{max}}\right)$$

Equ 28

In this case S_α could represent the specific entropy of a photon and S_{max} could represent the maximum specific entropy that can be associated with a single photon – i.e. the entropy that could be directly associated with the single photon itself. I am not at all sure what that means, physically, or even if such a concept is valid. **I need to read about "specific entropy", as it applies to photons, and otherwise.** Possibly the more meaningful interpretation would be entropy associated with the energy transported to the site of its capture for photosynthesis, within a certain time period. However, the amount of energy would be dependent on time duration, number of photons, and wavelength. Surely the grade of energy in a photon would not change due to duration of flux, or concentration of photons, so I am inclined to think that what I decide it means at the macro level has direct implications for the "specific entropy" of the photons themselves. So, for now, let me assume it has meaning, and G_α is the quality of the incoming energy E, and S_α is the contained entropy that is being brought into the system in this flux of high-energy low-entropy short-wavelength photons.

Such thoughts of "specific entropy" seem to imply to me a kind of duality of existence, vaguely similar to the wave/particle duality of light photons themselves. In the case of specific entropy, there is an implication that entropy can be localized in a single photon, can be carried from place to place, and can be injected into a system as low entropy units, or removed from a system as high-entropy units. I suppose this is why some people prefer to talk about negentropy, so it flips the meaning, and units of high negentropy are injected, and subsequently degrade, even as the energy degrades along with it. However, in many of the equations re entropy there is no localization implied, and, in fact, it seems that one must imply non-localization of entropy (e.g. heat reservoirs of enormous capacity and spatially homogenous temperature) in order to apply the formulae. So, we have a local/non-local phenomenon. I see this also when I read about Shannon's entropy in information theory. "Surprisals" (e.g. p_i x ln(p_i) is called a surprisal) are fragments of entropy that are added together to compute the overall entropy (see equation 06).

When I talk about a "flux" I imply that there is a passage of time. There are two sorts of such time durations that I need to think about. There is the duration of the flow of photons onto plant cells, and there is the duration of the process of photosynthesis. The first might last a few hours. The second might require only a fraction of a second, per photon. I think the second one is not important. I can work with the first, and then pro-rate it down to smaller size if I need it. So, I will assume the light shines on the plants for 12 hours, and then energy that was stored in the α-state was E. In a steady-state system the flux of energy out would equal the flux of energy in, so assumptions about inflows have direct implications for outflows.

Combining equations 27 and 28, the change in grade of the energy, as it flows through the trophic chain, is:

$\Delta G_{(\alpha\to\omega)} \equiv \left(1-\frac{S_\omega}{S_{max}}\right)-\left(1-\frac{S_\alpha}{S_{max}}\right) = -\frac{S_\omega - S_\alpha}{S_{max}} = -\frac{\Delta S_{(\alpha\to\omega)}}{S_{max}}$	Equ 29

Where all entropy is non-localized, i.e. associated with total energy E, the sum of all four bands of energy, e.g. ${}_E S_{max} = \sum_{i=1}^{4}\left({}_{e_i}S_{max}\right)$, etc.

I can also express the overall change in entropy using the P_{up}/P_{down} ratio from equation 07:

$\Delta S_{(\alpha\to\omega)} = \ln\left[\frac{P_{Up}}{P_{Down}}\right] = \ln\left[\frac{\pi(\alpha\to\omega)}{\pi(\omega\to\alpha)}\right]$	Equ 30

Combining equations 29 and 30 I get:

$\Delta G_{(\alpha\to\omega)} \bullet S_{max} = -\ln\left[\frac{\pi(\alpha\to\omega)}{\pi(\omega\to\alpha)}\right]$	Equ 31

I must note that S_{max}, here, is as per my Ref B definition, and not as per Truls Gundersen's definition.

Finally, I can make a table of transition formula, showing ΔS and ΔG for each transition.

Transition Name	Transition Event	Change in Entropy	Change in Grade	Equ #
(α→II)	e_1 degraded	${}_{e_1}\Delta S_{(\alpha\to II)} = \ln\left[\frac{\pi(\alpha\to II)}{\pi(II\to\alpha)}\right]$	$\Delta G_{(\alpha\to II)} = -\left[\frac{{}_{e_1}\Delta S_{(\alpha\to II)}}{S_{max}}\right]$	Equ 32
(II→III)	e_2 degraded	${}_{e_2}\Delta S_{(II\to III)} = \ln\left[\frac{\pi(II\to III)}{\pi(III\to II)}\right]$	$\Delta G_{(II\to III)} = -\left[\frac{{}_{e_2}\Delta S_{(II\to III)}}{S_{max}}\right]$	Equ 33
(III→IV)	e_3 degraded	${}_{e_3}\Delta S_{(III\to IV)} = \ln\left[\frac{\pi(III\to IV)}{\pi(IV\to III)}\right]$	$\Delta G_{(III\to IV)} = -\left[\frac{{}_{e_3}\Delta S_{(III\to IV)}}{S_{max}}\right]$	Equ 34

(IV→ω)	e_4 degraded	${}_{e_4}\Delta S_{(IV\to\omega)} = \ln\left[\frac{\pi(IV\to\omega)}{\pi(\omega\to IV)}\right]$	$\Delta G_{(IV\to\omega)} = -\left[\frac{{}_{e_4}\Delta S_{(IV\to\omega)}}{S_{max}}\right]$	Equ 35

These are all consistent with the intuitive idea that grades should be additive, and since ΔS and ΔG are proportional, the assumption that entropy is additive is somewhat confirmed.

I would like now to find a formula for a segment of the trophic chain, similar to equations 21 and 26. Consider a segment with k energy (or capital) transformations involving k+1 types of energy store (e.g. species, or businesses in a supply chain), enumerated by i where 1 ≤ i ≤ k.

$\Delta G_{(segment)} \bullet S_{max} = -\ln\left[\prod_{i=1}^{k}\left(\frac{\pi(i\to i+1)}{\pi(i+1\to i)}\right)\right]$	Equ 36

Where:

- $\Delta G_{(segment)}$ is the change in grade of the energy in the chain over the segment of the trophic chain;
- $\pi(i \to i+1)$ is the probability that transition i will happen, and the system will transit from state i to state i + 1;
- $\pi(i+1 \to i)$ is the probability that transition i will happen in reverse, and the system will transit from state i + 1 to state i;
- S_{max} is the idealized maximum entropy possible in the transition from α-state to ω-state.

4.6 - Problems

These arguments, leading to equations 21, 26 and 36 are the best I can do at the moment, but they raise more questions than they answer. For example:

- How do I develop a connection between what Truls Gundersen refers to as S_{max} and what I refer to as S_{max}? I suppose I would need to add new ABM in EiLab that has no upper limit on wealth (that would be Model K), and then redo my Ref B/F study of entropy production in that more general environment. Then I would be able to directly compare his "constant energy" concept of S_{max} with a new less constrained version of my S_{max}. This would NOT be a small exercise.
- Can photons actually carry specific quantities of entropy into a system, or out of a system? If yes, can this be quantified? If it can be quantified, would this enable interpretation of equations such as Equ 36? If I google "entropy in photon of light" there are many sites that come up. I need to read into that.
- Is there any way to express efficiency in terms of grade? I.e. can I find a way to combine equations 21 and 36? It seems to me there is an obvious connection between efficiency and degradation, but I cannot get there.
- What does equation 36 mean in the context of a trophic chain? A chemical transition can reverse, by low-probability transition in reverse, but a predator cannot un-eat some prey, nor can a business un-waste or un-purchase a lot of goods or a package a services (whatever interpretation I might try to put on a transformation of capital in an economy). This problem is, perhaps, inherent in the assumption by England (at Ref D) that thermodynamic arguments

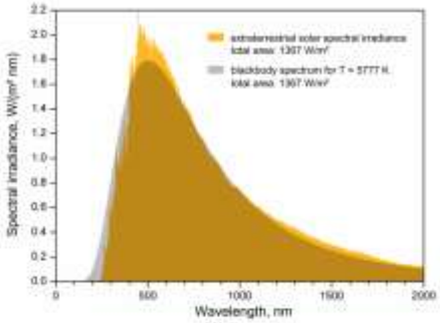
about molecules can be extended to super-macro phenomena such as replication of bacteria, etc. I have taken that assumption one step further, e.g., to apply it to consumption of one bacterium by another.

- What about webs? Trophic chains do not exist alone, but exist as a part of a trophic web, so these three equations might need to be generalized to apply to a web in place of a chain, if they are to be useful for anything in practice.
- How would you test these empirically? I could probably design an agent-based model that would demonstrate each, and so provide supporting evidence that the math is valid, but without addressing the above questions first, it would not be convincing. Or, a person could measure efficiencies of particular transitions (probably done already) and particular segments. But I have no idea how one might test equation 36 empirically in a real trophic web.

4.7 - Additional Information Re Entropy of Photons

The Wikipedia articles at Refs S, T, U and V describe black body radiation, effective temperature, the Stefan-Boltzmann law, Planck's law, and Wien's displacement law. All of these are relevant to the alpha and omega states of the energy as it flows from Sun to Earth (alpha radiation), and then from Earth to space (omega radiation). These relate as follows: All celestial bodies (stars and planets) radiate with a spectrum that has a distribution described by Planck's Law, and that distribution is determined by surface temperature. For a star, we work with effective surface and effective temperature, since the actuals are virtual. Wien's law relates temperature with peak wavelength. Kirkaldy provided (from Blum) information about peak wavelength for incoming and outgoing light for the Earth. From peak wavelength I should be able to compute effective temperature of incoming and outgoing light, and work with those numbers.

Figure 04 – Effective Temperature



The effective temperature of the Sun (5777 K) is the temperature a black body of the same size must have to yield the same total emissive power.

At Ref S the "Effective temperature" is discussed. Assuming that the radiation of a planetary or stellar object conforms to the distribution of frequencies called "black body radiation", then the effective temperature of the surface of the celestial object can be calculated.

I think what I need is the "effective temperature" of the radiation from the Sun just before it impinges on the surface of the Earth. So, rather than figuring the radius of the Sun (a tricky business, at best, for a ball of gas) I wonder if I can simply consider the radius of the Sun to be equal to the distance from Sun's core to the top of the Earth's atmosphere, and then compute the effective temperature there. Hmm?

The Stefan-Boltzmann constant is:

$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.670373 \times 10^{-8}\, Wm^{-2}K^{-4}$	Equ 37

The total (bolometric) luminosity (L) of a star with radius R is given by the Stefan-Boltzmann law:

$L = 4\pi {R_1}^2 \sigma \left(T_{eff}^4\right)_{R_1}$	Equ 38

Where R_1 is the radius of the star, σ is the Stefan-Boltzmann constant, and $\left(T_{eff}^4\right)_{R_1}$ is the fourth power of the effective temperature of the star. I have added the subscript R_1 on the temperature to distinguish it from the new temperature I propose to calculate. The Ref S document points out that figuring out what R_1 is can be a tricky issue. That does not concern me because the value of L (total output of the Sun) is what I think I need. Then, let R_1 be the radius of the Sun. Let R_2 be the distance from the core of the Sun to the surface of the Earth. Applying the same formula for the two radiant spheres, I get:

$4\pi {R_2}^2 \sigma \left(T_{eff}^4\right)_{R_2} = 4\pi {R_1}^2 \sigma \left(T_{eff}^4\right)_{R_1}$	Equ 39

Resolving for $\left(T_{eff}^4\right)_{R_2}$ I get:

$\left(T_{eff}^4\right)_{R_2} = \frac{4\pi {R_1}^2 \sigma}{4\pi {R_2}^2 \sigma} \left(T_{eff}^4\right)_{R_1} = \frac{{R_1}^2}{{R_2}^2} \left(T_{eff}^4\right)_{R_1}$	Equ 40

Then:

$\left(T_{eff}\right)_{R_2} = \sqrt{\frac{R_1}{R_2}} \left(T_{eff}\right)_{R_1}$	Equ 41

In words, the effective temperature of the Sun's radiation on the surface of the Earth can be calculated from the standard value of the effective temperature of the surface of the Sun (from the perspective of black body radiation) by a simple linear factor.

According to Kirkaldy (Ref T), when referencing Blum (1951) the effective temperatures in the Earth's entropy balance are:

- Incoming rays – distribution with maximum wavelength of λ_{max} = 4,800 Angstrom; and
- Outgoing rays – distribution with maximum wavelength of λ_{max} = 10,000 Angstrom.

Planck’s law describes the distribution of black body radiation, which is seen in Figure 04.

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\left[e^{hc/\lambda k_B T} - 1\right]}$$

Equ 42

This would be read as “The number of photons emitted with wavelength λ at temperature T is calculated using this formula.” From this, using numerical evaluation of an intractable mathematical phrase, we get Wien’s Displacement law, which can be used to relate the peak value for lambda (λ_{max}) to the effective temperature T.

$$\lambda_{max} = \frac{2.89776829 \times 10^6 \, nm \cdot K}{T}$$

Equ 43

I should then be able to calculate $\left(T_{eff}\right)_{R_2}$ in two different ways and see if they correlate:

- One, using the idea based on the Stefan-Boltzmann equation, i.e. equation 41.
- The other would be converting the 4,800 Angstrom wavelength into an estimate of effective temperature using Wien’s Displacement law, using equation 43.

4.7.1 - $\left(T_{eff}\right)_{R_2}$ Based on Stefan-Boltzmann Equation

I am using equation 41. For this I need three empirical values:

- R_1 = the effective radius of the Sun as used in calculating the surface temperature and black body radiation. When calculating the effective temperature of the Sun a number for the Sun’s radius is used, but I am having difficulty finding out what that is. It is the radius for which the Rosseland Optical Opacity is equal to 1. There are lots of sites that quote the Sun’s radius, but none indicate if this is the same radius as used for calculating effective surface temperature. At Ref W it says that the nominal solar radius is $R_1 = 6.957 \times 10^8$ m. This is the standard number used by astronomers to describe other stars, and, as such, is only an approximate number. It certainly is not the radius to be used for calculating the effective surface temperature.
- R_2 = the distance from the Sun’s core to the surface of the Earth, commonly called an Astronomical Unit (AU). At Ref X it says that one AU is now defined as R_2 = 149,597,870,700 meters. This is the average distance from Sun to Earth.
- $\left(T_{eff}\right)_{R_1}$ = the effective temperature of the surface of the Sun. At Ref S it says that the effective temperature of the Sun is $\left(T_{eff}\right)_{R_1}$ = 5780 K.

Equ 44

$(T_{eff})_{R_2} = \sqrt{\frac{R_1}{R_2}}(T_{eff})_{R_1} \cong 394\ K$	

4.7.2 - $(T_{eff})_{R_2}$ Based on Wien's Displacement Law

One Angstrom = 0.1 nanometer. From Kirkaldy,

- λ_{max1} = 4,800 Angstrom = 4,800 x 0.1 nm = 480 nm.
- λ_{max2} = 10,000 Angstrom = 10,000 x 0.1 nm = 1,000 nm.

$(T_{eff})_{R_1} = \frac{2.89776829\ \times\ 10^6\ nm\ \cdot K}{480\ nm} = 6037\ K$	Equ 45
$(T_{eff})_{R_2} = \frac{2.89776829\ \times\ 10^6\ nm\ \cdot K}{1{,}000\ nm} = 6037\ K$	Equ 46

4.7.3 - Entropy Per Photon

But, then, the question would be whether this could then be fed into Clausius' definition of entropy. I need to make a lot of assumptions here, and a lot of simplifications. Consider a flux of N_{in} incoming photons, each carrying an amount of energy equal to hc/λ. Most of these photons would not be able to participate in photosynthesis, and so would not be relevant to this discussion.   The visible light is in the range of peak wavelength, and this is also the range that photosynthesis works with (I think) so I can assume the rate of energy takeup via photosynthesis is determined by the most common wavelength in Sunlight, i.e. by λ_{max}. Then the total energy in the flux is $Q_{in} = N_{in}$ hc/λ having entropy (in Clausius' terms) of $S_{in} = Q_{in} / T_{in}$.

Then, the entropy carried in the incoming photons that are destined to be absorbed via photosynthesis would be:

$S_{in} = \frac{Q_{in}}{T_{in}} = \frac{N_{in} \times hc}{\lambda_{max} \times T_{in}}$	Equ 44

The logic around the outflow of energy needs to be a little different. The incoming light is filtered by the photosynthesis process, so the incoming flux is in a narrow band of frequencies (of wavelengths) that are appropriate. However, the outflow is not filtered, and is mixed with the outflow of all of the other energy that was absorbed by the Earth, degraded (e.g. by weather systems) and eventually emitted. So the maximum wavelength of the outgoing radiation spectrum defines a full black body spectrum. But, the outgoing energy of interest to me is the same as the incoming (filtered) energy band. It comes in as a narrow band of synthesizing radiation, and goes out as a broad band of black box radiation.

I can use the Blum/Kirkaldy λ_{max} and Wien's displacement law to calculate T_{out}. (Or, I could look up the effective temperature of the Earth's surface. I could then use the Stefan-Boltzmann law to calculate the total energy flux out. Then the outgoing entropy would be $S_{out} = Q_{out} / T_{out}$, where $Q_{out} = Q_{in}$. Wait?!? Is it as easy as that? Hmm? Perhaps I don't need all the fancy logic around the outflow. Then the change in entropy would be:

$$\Delta S = S_{out} - S_{in} = \frac{Q_{in}}{T_{out}} - \frac{Q_{in}}{T_{in}}$$

Equ 45

If S_{max} is defined as Q_{out} / T_{bg} where Tbg is the effective temperature of the background radiation in the universe, then the change in grade of the involved energy would be:

$$\Delta G = \frac{-\Delta S}{S_{max}} = \frac{T_{bg}}{T_{out} - T_{in}}$$

Equ 46

That doesn't look quite right. It looks upside down. CHECK IT!

5 - Summary

I have achieved my goal of combining equations from several sources to get some insight into how energy is degraded, and how degradation steps down as time moves forward. However, it is still somewhat less than satisfying, due to the range of questions it raises.

6 - Loose Ends?

Next steps for this note would be:

- Read more about specific entropy, and especially entropy associated with photons.
- Try to answer some of the questions raised.
- Contact Truls Gundersen to ask if he could provide some guidance.
- Think through how this would apply to a supply chain and the degradation of capital.
- ??

On page 969 of Ref T (Kirkaldy, 1965) the author presents this formula:

$$\frac{dS}{dt} = \frac{\vec{J_q}}{T_i} - \frac{\vec{J_q}}{T_o} + \frac{d_i S}{dt}$$

Equ 37

Where:

- $\vec{J_q}$ is the heat inflow or outflow, into or out of the biosphere. This would be the energy carried in the photons in the alpha-state or the omega-state.

- T_i is the “effective temperature” of the incoming photons. Blum (1951), see Ref R, describes this radiation as having a distribution with a maximum wavelength of 4,800 Å.
- T_o is the “effective temperature of the outgoing photons. Again, Blum describes this radiation as having a distribution with a maximum wavelength of 10.000 Å.
- d_iS/dt is the rate of entropy production within the system (the biosphere) due to irreversible processes.

Kirkaldy does not tell us what the effective temperatures are, or how to calculate them. Nevertheless, this equation succinctly captures the intention of my approach. This needs (a) a more recent estimate, and (b) a more detailed explanation.

7 - Annex A – Discussion with Mark xxx on Quora

7.1 - References

A. J.S. Kirkaldy, "The Thermodynamics of Terrestrial Evolution", (1965)
B. Quora – a forum for discussion – (https://www.quora.com/Can-we-quantify-the-change-in-entropy-of-the-Earth-when-a-photon-arrives-in-or-leaves-the-atmosphere)
C. Mark Barton, PhD in Physics, The University of Queensland, physicist with National Astronomical Observatory of Japan.
D.

7.2 - Boyle to Quora

Can we quantify the change in entropy of the Earth when a photon arrives in or leaves the atmosphere?

7.3 - Barton to Boyle

No, because it depends on what else is going on. If the photon arrives as part of a highly collimated, monochromatic beam, as from a laser, then it's highly ordered and equivalent to work for the purposes of thermodynamics, so in principle you could convert it to mechanical work with 100% efficiency.

Conversely, if it arrives as Black-body radiation with some particular temperature (for example and in particular, solar energy with an effective temperature of some 5500 K), then it's maximally disordered for its energy density and equivalent to heat with that temperature.

And in either of these cases, you could squander whatever negentropy it had by letting it hit something (like a rock) at some typical ambient temperature like 20°C.

7.4 - Boyle to Barton

Mark.
Thanks. That's helpful.
But, let me refine the question slightly. What if (1) the photon arrives as black body radiation from the Sun and is captured by the biosphere via photosynthesis, or (2) is emitted by the Earth as black body radiation? For either of those, can the change in entropy of the Earth be quantified? Are there published papers that address these sorts of questions?

7.5 - Barton to Boyle

If it's solar radiation then it brings entropy equal to its energy divided by the temperature of the photosphere of the sun, about 5500 K, which is quite modest, because 5500 is a large number. So if you put the light into a solar concentrator and maintain a heat reservoir at 5500 K or thereabouts, you can in principle run a Carnot cycle heat engine off it and get mechanical work or other ordered energy out of it with relatively high efficiency. And that's the best you can do.

However if you just let it warm a rock to say 20 °C = 293 K, then it adds entropy equal to its energy divided by 293 K, which is larger because 293 is a smaller number. So by using the light wastefully you've generated entropy. Fortunately you're not quite screwed because the night sky is colder again.

And solar electricity and photosynthesis have somewhere between the upper limit set by the temperature of the sun and the temperature of the night sky, and zero, set by squandering all the opportunities and letting the heat drain away to space.

7.6 - Boyle to Barton

Mark.

That’s good news!

I am trying to understand the biosphere from the perspective of a Carnot heat engine. So, if I understand you correctly, I could estimate the energy captured via photosynthesis then divide by the effective temperature at Sun’s surface (as in Clausius’ Q/T). Theoretically, I could get the energy by multiplying [flux of photons per second captured by photosynthesis] times [energy per photon] integrated across [the energy band that participates in photosynthesis (Photosynthetically active radiation - Wikipedia)]. Then delta entropy per photon could be calculated, however modest it may be.

I would expect it to be “modest” as the arriving energy would need to increase Earth’s entropy by a very modest amount, and the departing infrared radiation would need to reduce Earth’s entropy by a significantly larger amount (per photon?). Both of those changes would have to compensate for the increased entropy caused by respiration within the biosphere, generating an entropy balance such as discussed by Kirkaldy (Thermodynamics of Terrestrial Evolution, 1965).

But, I was wondering about what number I should use for radius of the Sun when calculating the effective temperature of the Sun’s surface: the normal radius of the Sun (giving an effective temperature of about 5500 K, as you say), or one AU (giving an effective temperature of about a tenth of that). I suppose that was my way of diluting the Sun’s effect (in place of a concentrating mechanism, as you suggest). I am beginning to suspect that approach abuses the meaning of “effective”.

But, perhaps, that is not needed. Perhaps using photosynthesis as a filter has a similar effect as your suggestion - some sort of solar concentrator. That filter collects almost monochromatic light, essentially selecting some order out of the disorder of black body radiation, and enabling some work to be done.

I appreciate your comments.

Garvin