A RESTATEMENT OF ODUM'S MAXIMUM POWER PRINCIPLE

Abstract

H. T. Odum was a free-thinking man of enormous talent whose academic career spanned almost 50 years. Among his many and varied contributions to the field of systems ecology is a concept he called the Maximum Power Principle. In 1955, H. T. Odum and R. C. Pinkerton proposed that the MPP is a presence in a wide variety of autocatalytic systems from physical, to organic, to ecological, to social, and even to economic systems. They proposed that it should be given status as the fourth law of thermodynamics, due to its wide-ranging occurrence, and its broad explanatory power. Nevertheless, despite the evidence that such a widespread and important phenomenon exists, their proposal has received remarkably little attention, apart from that given by Odum's students and close colleagues. We speculate that this is due, in part, to the complexity of the concept, in part, to the difficulties it presents to researchers, and in part, to the idiosyncratic diagrams and language developed and used by Odum over the course of his 50-year career. This paper revisits the early formulation of the concept, recaps its history, restates the principle as a series of general falsifiable postulates, and proposes a means by which those postulates can be tested.

SECTION I – Introduction

H. T. Odum, and R. C. Pinkerton have proposed that a phenomenon herein called the maximum power principle (and referred to as the MPP) be considered as a candidate for the fourth law of thermodynamics. That proposal has received little consideration in recent years, and we believe that, with present advances in the study of open dynamic systems, it deserves a fresh examination.

The purpose of this article, then, is to restate the MPP, in a general form, as a series of falsifiable hypotheses, and thereby make it accessible for validation or repudiation. In section II of this paper, we review the conceptual origins of the MPP and outline the history of the literature that has addressed it, starting with a paper by A.J. Lotka. In section III we examine in some detail two distantly related phenomena that are drawn from physics, and which provide significant insight into the inner dynamics of the phenomenon. While Odum believed that the principle is widely applicable in physics, in thermodynamics, in chemistry, in biochemistry, in ecological studies, and also in economics, these two examples from the physical sciences are relatively easy to treat mathematically, and so provide an easy portal into an deeper understanding of the peculiar dynamics at the heart of the phenomenon. In section IV we outline a variety of concepts and terminology, the understanding of which is a necessary precursor to a clear restatement, in modern language, of the MPP. In Section V we present our restatement of the MPP as a series of falsifiable hypotheses, and include a discussion of some of the implications of each component hypothesis. Then, in section VI we discuss issues around validation or falsification of the MPP. Finally, in section VII we summarize our arguments.

SECTION II – Conceptual Origins and History

In 1922, A. J. Lotka [1] described what he referred to as the ***principle of maximum energy flux***. His paper was written as part of an ongoing discussion in the contemporary literature on the topic of the rate of entropy production in ecosystems. He began his argument drawing on a variety of texts by Ludwig Boltzmann:

> *"It has been pointed out by Boltzmann that the fundamental object of contention in the life-struggle, in the evolution of the organic world, is available energy. In accord with this observation is the principle that, in the struggle for existence, the advantage must go to* ***those organisms whose energy-capturing devices are most efficient*** *in directing available energy into channels favorable to the preservation of the species".*

In this paragraph Lotka is noting the then dominant view that evolution leads to increased efficiency of energy capture of organisms. Lotka did not challenge this idea at the level of the species, but, in a series of arguments, he rapidly expanded his focus from organisms and species to include the entire ecosystem, and consider the effects of natural selection in that wider arena, ultimately summarizing his arguments in the following paragraphs:

> *"In every instance considered, natural selection will so operate as to increase the total mass of the organic system, to increase the rate of circulation of matter through the system, and to increase the total energy flux through the system, so long as there is presented an unutilized residue of matter and available energy.*
>
> *This may be expressed by saying that natural selection* ***tends to make the energy flux through the system a maximum****, so far as compatible with the constraints to which the system is subject."*

In the above excerpts, the emphasis (in bold) is ours. Note that Lotka proposes that, at least at the level of systems, as opposed to organisms, the maximization of energy fluxes is the expected outcome of prolonged evolution, and not the maximization of efficiencies of energy capture. In a significant footnote to his arguments he defines "energy flux" as "the available energy absorbed by and dissipated within the system per unit time".

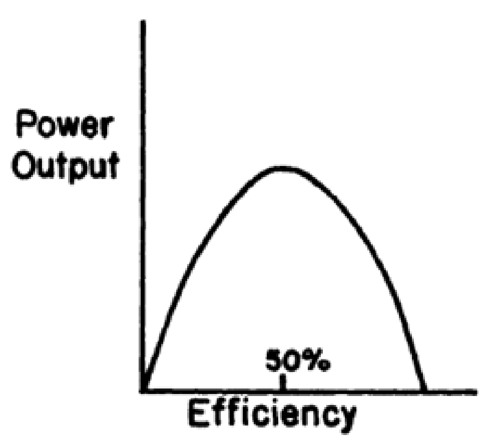
In 1955, H. T. Odum and R. C. Pinkerton [2] revisited Boltzmann's and Lotka's ideas and framed them in terms of the relevant developments that had occurred in thermodynamic theory in the intervening years. To introduce their input on the issue, we can do no better than to here copy the introductory paragraphs from that ground-breaking paper of 1955:

> *AMONG THOSE who deal with the many separate sciences, and among those who seek universals common to the various sciences, there is a search to find out why the thousands of known processes are regulated, each one at a characteristic rate. A common denominator has been found in the concept of entropy which permits the comparative study of energy changes. For closed systems natural spontaneous processes are directed toward an entropy increase, so that entropy has been appropriately called "time's arrow." What has been lacking, however, is a generalization applicable to open systems which would indicate the rate of entropy increase. The Second Law of Thermodynamics does not indicate the magnitudes of the rates or explain how open*

systems are adjusted. If it exists, we need to discover "time's speed regulator." Theories of rate processes are available for simple molecular scale systems, based largely on statistical thermodynamics, but such detailed pictures are impractical in complex systems. In the theory of automatic controls (servo-mechanisms), in economics, and in other fields, another approach may be used involving the assumption that rates in question are proportional to the forces causing them. In order to understand rate adjustments in living systems, additional hypotheses are required. In this discussion a simple general expression for idealized systems is presented relating efficiency and power. ***Our proposition is that natural systems tend to operate at that efficiency which produces a maximum power output.*** *With expressions derived from this basic assumption, it is possible to distinguish between the maximum power hypothesis and alternative propositions, e.g. the supposition often made that systems tend to run at maximum efficiency. One of the vivid realities of the natural world is that living and also man-made processes do not operate at the highest efficiencies that might be expected of them. Living organisms, gasoline engines, ecological communities, civilizations, and storage battery chargers are examples. In natural systems, there is a general tendency to sacrifice efficiency for more power output. Man's own struggle for power is reflected in the machines he builds. In our energy-rich culture, most of our engines are designed to give maximum power output for their size.*

Utilizing ideas derived earlier, Lotka (1922) proposed a "law of maximum energy" for biological systems. He reasoned that what was most important to the survival of an organism was a large energetic output in the form of growth, reproduction, and maintenance. Organisms with a high output relative to their size should win out in the competitive struggle for existence. Let us make the following postulate: ***Under the appropriate conditions, maximum power output is the criterion for the survival of many kinds of systems, both living and non-living.*** *In other words, we are taking "survival of the fittest" to mean persistence of those forms which can command the greatest useful energy per unit time (power output). Holmes [3] discusses the historically independent postulates such as that mentioned above which attempt to predict general trends for open systems. Although there are probably many situations where power output is not at a premium, let us in this discussion consider that type of energetic coupling which does produce a maximum power output.*

Figure 1 – A Power-Efficiency Curve



Power as a function of efficiency, taken from the paper by Odum and Pinkerton, 1955.

A simple process involving an energy transfer can be considered as a combination of two parts. In one direction, there is a release of stored energy, a decrease in free energy, and the creation of entropy. In the other direction, there is the storing of energy, the increase of free energy, and an entropy decrease. The whole process consists of a coupling of the input and output. The Second Law of

Thermodynamics requires only that the entropy change for this "open" system and its surroundings be a simple increase in order for the over-all process to occur. It is possible for the input and output to be coupled in various ways so as to produce varying rates and efficiencies.

We draw your attention to the two key propositions shown in bold (our emphasis) in the above quoted passage. Note that, in reference to and in accordance with Lotka's paper, Odum and Pinkerton expand their focus from the energy consumption and persistence of individual organisms and species up to the energy consumption and persistence of systems. Having thus expanded the scope of their consideration, and presented their generalized propositions, in following passages they then narrow their focus again from the natural system as a whole to instances of energy transfer within the system, and they then go one step further to consider a specific class of such energy transfers which can be described with a pair of thermodynamic force-flux equations based, in part, on considerations of Onsager's reciprocal relations. [4, 5]

But, just before presenting those force-flux equations and embarking upon the intricate and arcane argumentation supporting a generalized thermodynamic principle in that specific context, they present the example of Atwood's Machine (herein referred to as the AM). The AM provides an example which is easily accessible because it describes energy transfers using the simple and widely-understood equations of Newtonian motion. They argue that energy transfers within natural systems exhibit a somewhat similar power-efficiency graph, as shown in Figure 1 (copied directly from the paper of 1955), for which the power is zero at efficiencies of either zero or one, but in which power rises smoothly to a maximum at some intermediate level of efficiency. As such, it forms a strictly concave map on the interval [0, 1].

The proposed principle as described in the paper of 1955 did initially receive some attention by other analysts:

- By 1961 the thermodynamic arguments, been revised slightly, were reproduced in the landmark work by Tribus. [6]
- In 1976 Smith [7] published a careful study of the applicability of the AM, as a dynamic model, to actual power-efficiency relationships found in nature. In his presentation of the dynamics of the AM, he points out an error in the paper of 1955, explained below. But having corrected the error, he proceeds to think through the premise of the argument in a number of cases, with a particular focus on the set of plant species within a sere of secondary terrestrial succession. He considers a number of uses of accumulated energy, such as individual growth for both primary energy capture tissues (e.g. leaves) and for support structures (trucks and branches), metabolic costs of capturing energy, and the diversion of energy to defence and reproduction. Notably missing from his discussion is the ultimate end of much of the energy accumulated for reproduction which is exploited by other species via predation and parasitism. In his conclusions he stated "*The trade-off between power and efficiency demonstrated by Atwood's machine is based on the thermodynamic principle that energy conversions cannot be both fast and efficient. The conversion that gives the most power is of intermediate efficiency.*" We note that his focus on succession of plant species indicates an interest in the evolution of ecosystems, but it is only a part of the broader concept that we believe Lotka, Odum and Pinkerton had in mind.
- The concept continued to stir up sufficient interest to be attacked six years later by W. Silvert, in 1982 [8], who outlined what he perceived to be a number of flaws in the paper of 1955, including the mathematical error mentioned by Smith. In particular, he argued that the

AM was a non-dissipative machine that could never effectively model the dissipative activities of an organism. But, curiously, having argued flatly that the premise of that paper was wrong, he found strong empirical evidence in support of its conclusions. That is, he stated that "*These errors do not vitiate the importance of their approach, and this paper presents an alternative ... approach ...* ", and he later concludes by saying "*There is empirical evidence that diverse classes of organisms exhibit the same ecological efficiency ...*". He adds "*it is plausible that the* ***concept of optimal efficiency*** *will prove so broadly applicable as to be a major contribution to ecological theory.*" (Note the name change. The emphasis is ours.)

- Fortuitously, this provoked a rebuttal by H.T. Odum in 1983 [9], in which he outlined his many reasons for continuing to believe the MPP to be an important, widespread, and little understood phenomenon. This is, perhaps, the most concentrated and most succinct summary of his opinions on this topic. He goes on in this paper to name a variety of other independent studies that were coming to similar conclusions, and to name a number of issues that warranted further thought, and further study.
- Here are three key points he makes that are relevant to this present exercise in which we will restate the MPP. First, he defended the use of the AM as a simple physical model, having obvious limitations, which nevertheless provided important insights into the dynamics of all energy transformations. Second, he mentions an additional postulate published in 1967 [10] to the effect that, after selection, surviving systems have organized their pathways and loading so as to minimize entropy tax at maximum loading. Here, the "entropy tax" is the energy that is wasted as useful energy is accumulated. We will be returning to this important concept later. Third, he discusses the applicability of the MPP to economic systems.

As stated in the opening paragraphs of his paper of 1955, Odum's life vision was to understand those dynamic phenomena that appeared in analogous forms in many varied and distinct natural systems, to formulate a common description of them, and to then exemplify their behaviour through application to ecosystems. Howard Thomas Odum (H.T.) and his brother Eugene Odum did much work together and founded the field of study now called "Systems Ecology". In modern terminology, we would say they were working to develop a general description of self-organizing complex adaptive systems. However, in those early years, many of the mathematical techniques now used to study such complex adaptive systems had not yet been developed. Indeed a wide variety of related concepts, such as Prigogine's theory of dissipative structures, catastrophe theory, chaos theory, and the mathematics of fractals, were in the process of being developed during those years. As it has been when any new field of scientific study is emerging, there were no well-developed systems language tools (e.g. mathematical tools) to express such concepts that cut across many disciplines such as physics, chemistry, biology, ecology, sociology and economics.

In this turbulent intellectual environment, the Odum brothers developed their own graphic language that they termed the "energy circuit language", or "energy systems language" (ESL), into which they could translate any concept about dynamic systems developed in any field of study, and through which they could study that concept in application to ecological systems. Because of the close analogy between flows of electricity in circuits, and flows of energy through ecosystems, the primary concepts tended to be drawn from the study of electricity. The language consisted of a highly refined set of symbols representing types of flows and stores of matter and energy, with a set of rules on how to link the symbols into energy circuits, how to reduce complex circuits into more simple circuits, and how to interpret circuit diagrams into English statements with a heavy reliance on terminology drawn from thermodynamics, and an understanding of Onsager's reciprocal relations.

In fact, in his magnum opus entitled "Ecological and General Systems" [11] Odum identified thirty-three different 'systems languages' in use by the academics of his day in various fields of study. In this text book of over 600 pages the maximum power principle is just one among the many kinds of dynamic interactions described in this work, though given some prominence. Unfortunately, the highly idiosyncratic nature of the ESL made it difficult to access for new students. Though designed to be applicable in all fields of study of dynamic systems, it was taught and used in very few, only taken up by those self-motivated students who chose to do so.

There is plenty of anecdotal evidence supporting the idea that a widespread autocatalytic phenomenon like the proposed MPP is active in all persistent natural systems. But, on the whole, the MPP did not seem to receive the serious attention that it deserved, and, outside of a few of Odum's closest associates, it seems to have garnered little attention in the published literature since Silvert's attack. We speculate that this is, in part, due to the complex nature of the concept, in part due to the arcane mathematics used to explain the MPP in the article of 1955, and, in part due to the highly idiosyncratic nature of the ESL used by Odum in later years. The concept was and remains inaccessible because the learning curve was and remains too steep. Odum nevertheless went on, over the remainder of his lengthy career as a systems ecologist, to put forward the idea that this fundamental dynamic phenomenon plays a key role in most, or possibly all, autocatalytic natural systems, and believed it would ultimately be recognized and take its place as a fourth law of thermodynamics.

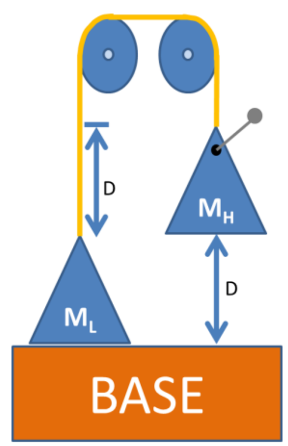
SECTION III – Related Phenomena

In this section we examine two physical phenomena which have both generated some controversy, but which provide some insight into some key concepts that comprise the MPP.

Atwood's Machine (the AM)

In 1784 George Atwood, an English mathematician, designed a type of machine which now bears his name – a machine intended to be used to study Newtonian laws of motion. (See Figure 2.) It is now used in many modern physics classes to demonstrate motion under constant acceleration due to gravity, and, it is also the proto-type for all modern machines that use counter-balancing weights, such as elevators or garage door openers.

Figure 2 – Atwood's machine



Two-pulley design of the AM.

It comes in many variations, but the one preferred for the remainder of this discussion has two masses joined by a rope hung over two identical pulleys. Conceivably, in a thought experiment, the two halves of the AM can be decoupled, and then coupled to other similar half AMs. Each half of the machine forms a device in which gravitational energy may be stored. The heavier mass (M_H) is held a distance D off the floor by a pin, and the lighter mass (M_L) rests on the floor or base of the machine. The pulleys and the pinning device are attached to a back board (not shown) that is attached to the base. To operate the machine, simply remove the pin, and the mass assembly (the two coupled masses) will accelerate smoothly as the heavier mass falls a distance D to the base and the lighter mass rises a distance D above the base.

It happens that this simple mechanical device exhibits a peculiar dynamic that is at the heart of Odum's proposed Maximum Power Principle, and an understanding of the peculiarities of that dynamic is key to understanding his perspective. In this machine have two coupled stores of energy (M_L and M_H) and a transfer of energy from one to the other. Let g represent the constant acceleration due to gravity. Let T represent the length of time it takes for the coupled-mass assembly to move as the mass M_H descends to the base. Consider three classifications of energy as the AM runs its course and the mass assembly moves:

Energy Type	Description of energy	Formula for Time-Averaged Power	Equation Number
Total	$E_T = M_H g D$ = The original endowment of gravitational potential energy stored in the pinned mass M_H.	$\overline{P_T} \equiv E_T/T = \frac{M_H g D}{T}$	Equ 01
Useful	$E_U = M_L g D$ = The quantity of gravitational potential energy ultimately stored in the lighter mass M_L.	$\overline{P_U} \equiv E_U/T = \frac{M_L g D}{T}$	Equ 02
Waste	$E_W = (M_H - M_L) g D$ = The residual of the original endowment of energy which is reversibly converted to the kinetic energy of the two-mass assembly, and is then irreversibly transformed into waste heat as the heavier mass collides with the base, and as the lighter mass bobs and trembles for a while, finally coming to rest at its final position. The irreversible transformation of this energy to waste heat is accompanied by a simultaneous rise in thermodynamic entropy in the AM and its environment.	$\overline{P_W} \equiv E_W/T$ $= \frac{(M_H - M_L) g D}{T}$	Equ 03

There are several characteristics of this dynamic that one should note. The power of interest is the time-averaged power over the interval [0, T], and not the instantaneous power at time T. Note that the wasted energy is referred to by Odum as the entropy tax or second-law tax. Also note that there is a small amount of energy loss not addressed in the above simple argument – an energy loss that might be referred to as an additional second-law tax – and that is the energy that is degraded due to friction. This energy is, of course, not truly "lost", as the first law tells us energy can never be destroyed. This energy is merely degraded, and converted into a form that cannot ever be used for the same purpose again, and escapes from the system into the environment around it as waste heat, waste heat, as the second law tells us may happen. It is prudent, temporarily, to distinguish between the second-law tax associated with the nature of the transfer of energy between the coupled stores of energy, and that second-law tax associated with friction. To simply maintain that distinction, we assume that the friction is zero.

So, setting aside, for the moment, this small amount of energy degradation due to friction, it is useful to consider the transformations that led to the still useful energy remaining in the lighter mass when the dynamic has run its course. Suppose that the entropy of the machine can be said to be localized in the coupled masses that form the mass assembly. As the dynamic runs its course that entropy rises, according to the second law of thermodynamics, and that rise in entropy is evidenced by the waste heat produced as the kinetic energy is degraded upon impact of the heavy mass with the floor. The ability of the machine to do work has been reduced in the process. But, rather than considering the mass assembly as a whole, think about the entropy that might be associated with each individual mass located in each half of the machine. At the beginning, the heavier mass is able to do work by falling to the floor, and the lighter mass which already rests on the floor is not. The heavier mass originally has low entropy, relative to the lighter mass. As the dynamic proceeds, a portion of the gravitational potential energy originally contained in the heavier mass is transferred to the lighter mass reversibly, thereby transferring some of the ability to do work to the lighter mass. When the dynamic has completed its course, the lighter mass now has reduced entropy (increased ability to do work) even as the heavier mass has increased entropy (decreased ability to do work). The energy thus transferred has retained its ability to do work as the entropy content of the lighter mass was decreased. At the same time the rest of the gravitational potential energy originally contained in the heavier mass is converted reversibly to inertial (kinetic) energy, in a fashion similar to a frictionless pendulum. That inertial energy is eventually converted to waste heat, but only when the mass assembly is forced to stop moving.

So, in the operation of this machine we see how the measure of entropy within a portion of a self-organizing system might be lowered as the self-organizing dynamic proceeds, even as the constraint expressed by the second law of thermodynamics is respected, and the overall entropy of the system plus environment rises.

We can define the efficiency of the AM as the ratio of the "useful" energy output divided by the "total" energy input. It is important to note that the useful energy is the energy which is stored for later use. The focus is not on the immediate consumption of energy, but, rather, on the effective transfer of energy for accumulation, and possible later use. This ratio is given by equation 4. Note that if M_H is less than M_L the AM will not function, so the efficiency of an operational AM necessarily has a value from the interval [0, 1].

$$\eta^{AM} \equiv \frac{E_L}{E_T} = \frac{M_L g D}{M_H g D} = \frac{M_L}{M_H} \qquad \text{Equ 04}$$

Using the Newtonian equations of motion we can solve for T and substitute the resultant expression into equation 2 to obtain the formula in equation 5.

$$\overline{\mathrm{P}_U} = M_L \left(\frac{D g^3 (M_H - M_L)}{2(M_H + M_L)} \right)^{1/2} \qquad \text{Equ 05}$$

If we define M_T as the sum of the masses:

$$M_T \equiv M_H + M_L \qquad \text{Equ 06}$$

then we can write M_H as:

$$M_H = M_T - M_L$$

Equ 07

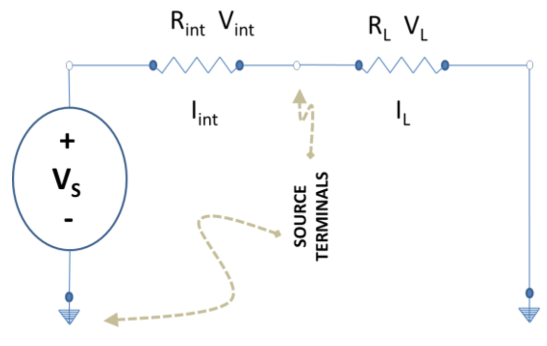
Curiously, when $\overline{P_U}$ is plotted against η^{AM}, with the constraint that M_T is held constant, we get a strictly concave map on the interval [0, 1], as shown in Figure 1.

Jacobi's Law

In the mid 1800s Moritz von Jacobi published a description of a phenomenon now known as Jacobi's Law, or the maximum power transfer theorem. He showed that, given a pre-existing electrical source in a simple circuit, any load connected to it will function at maximum power only if the resistance of the load matches the resistance of the source. Students of electrical engineering also know this phenomenon as resistance matching, or impedance matching.

Figure 3 shows a standard simple circuit that demonstrates Jacobi's law. V_S is the source voltage and R_{int} is the internal resistance of the source. R_L is the resistance of the external load.

Figure 03 – A Simple Circuit



While this example lacks a key component of what might be considered a standard energy transfer associated with Odum's maximum power principle, i.e. the re-storage of useful not-yet-degraded energy in a second energy store, nevertheless, this simple electric circuit exhibits a dynamic similar to that of the AM – similar, but not the same. Let T be some arbitrary finite time for which the current in the circuit is constant over the period [0, T]. We can define power in three ways as the circuit operates:

Energy Type	Description of energy	Formula for Time-Averaged Power	Equation Number
Total	$E_T = I^2(R_{int} + R_L)T$ = The energy released from the source over the duration [0, T], i.e. the total energy expended during the operation of the circuit.	$\overline{P_T} = I^2(R_{int} + R_L)$	Equ 08
Useful	$E_U = I^2 R_L T$ = The energy degraded by the load resistance, R_L, over the duration [0, T], i.e. the energy expended in the performance of useful work.	$\overline{P_U} = I^2 R_L$	Equ 09
Waste	$E_W = I^2 R_{int} T$ = The energy degraded by the internal resistance of the source, V_S, over the duration [0, T], i.e. the energy wasted.	$\overline{P_W} = I^2 R_{int}$	Equ 10

Note again that the power of interest is the time-averaged power over the interval [0, T], and not the instantaneous power at time T. This example differs from the AM in a couple of ways that must be noted. First, in this case, the time-averaged value for power and the instantaneous value are the same, but one should keep the distinction in mind. Second, the energy associated with the load is not stored for later use, but, rather, is immediately degraded by the load. The addition of an energy storage mechanism such as a capacitor introduces complicating factors associated with impedance. The addition of a storage for gravitational potential energy, say by using a motor to lift a mass, introduces complicating factors associated with the efficiency of the motor, and losses due to friction. For these reasons Jacobi's Law provides a less clear, and less instructive example, but it nevertheless does offer some insight as a simple alternative example, and so is included here.

We can define the efficiency of this circuit as the ratio of the energy expended for "useful" work divided by the "total" energy expended, as shown in equation 11.

$$\eta^{JL} \equiv \frac{I^2 R_L T}{I^2 (R_{int} + R_L) T} = \frac{R_L}{(R_{int} + R_L)} \qquad \text{Equ 11}$$

If we define R_T as the sum of the resistances:

$$R_T = R_L + R_{int} \qquad \text{Equ 12}$$

then we can write R_L as:

$$R_L = R_T - R_{int} \qquad \text{Equ 13}$$

Curiously enough, again, if we plot $\overline{P_U}$ against η^{JL} we again get a strictly concave map on the interval [0, 1], as shown in Figure 1.

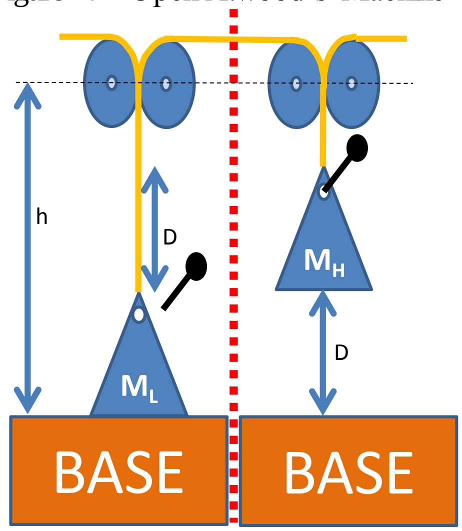
The AM and Jacobi's Law, And Necessary Features of Persistence

In Odum's vision of the maximum power principle, he saw energy being transferred from molecule to molecule down a metabolic pathway, or from organism to organism within a trophic web. The effective regular transfer of still-usable energy from store to store is a necessary feature of every persistent energy processing pathway.

The AM only fits into this vision partially, as explained by Smith. [7] When faced with an AM primed and ready to go, the AM already has an endowment of energy in M_H. We have not discussed the process that might endow it. There is a transfer from store to store. But, again, we did not discuss the process that then might pass that energy on to a subsequent energy store in a chain or web. So, while the AM does effectively model the transfer of energy from one store to another store, it is not done in such a way that we can easily envisage a pathway of stores down which energy can pass. To imagine such a pathway we would have to be able to decouple the two halves of the AM, and recouple them to other similar partial devices. This is clearly not physically possible, and can only be done as a thought experiment.

So, here is an interesting thought experiment. Imagine an AM for which energy can flow in as well as out, and call it an open AM, or OAM. Imagine that the two halves of this OAM can be delinked, and each half relinked to another such half of an open AM. Denote a half of an open AM as a HOAM. Suppose we have a collection of such HOAMs, and suppose the height "h" of the pulleys is very large compared to the distance D. (See Figure 4.) One can then imagine a system in which large amounts of energy are somehow captured by those HOAMs having the larger masses, and made available to the other HOAMs with smaller masses. In this system its HOAMs autonomously link and transfer that energy down pathways or chains of linkages, separating and degrading a portion of the energy at each step as it progresses. Finally, one can imagine that the HOAMs with the smallest masses, being at the receiving end of many transfers, can accumulate a great deal of energy as their mass is raised again and again, reducing their internal entropy during each linkage event. At some point the mass in the HOAMs will all be raised to the top, no longer able to link, and as the flow of energy starts to back up, the mid-sized HOAMs will also fill up. Eventually the system of HOAMs will have achieved an internal state of ultra-low-entropy.

Figure 4 – Open Atwood's Machine



An imaginary "open AM" (OAM) consisting of two linked Halves of Open AMs (HOAMs).

It is possible to produce an analytic formula for the power-efficiency function for at least three different types of constraint on the design variations for the AM. Two of these form strictly concave curves, and one does not. In computer simulations, it has been found that the two curves that are strictly concave appear in persistent simulations, and the third does not lead to persistency. (See equations 14, 15 and 16 below.)

Figure 04 – A Model Persistent Pathway with Persistent Linkage Events

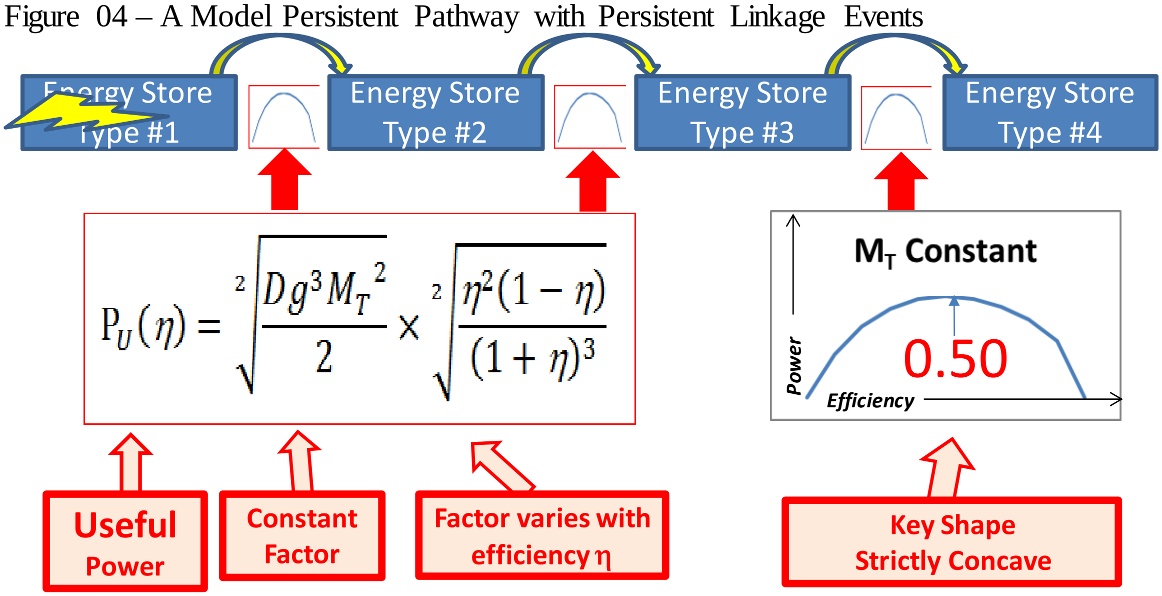


Figure 04 is a pictorial model of a persistent pathway based on our computer model. In the computer model the dynamics of the AM were used to model the linkage events, where indicated by the strictly concave curves. The formula for the useful energy transferred at each linkage event consists of a situational constant times a factor that is dependent only on the efficiency of the transfer. In the model a trophic web evolved with four or more trophic levels of organisms (types of stores) representing autotrophs, omnivores, carnivores and apex carnivores. Linkage events (i.e. predation events) exhibited a range of efficiencies on a continuum from 0.25 to 1.00.

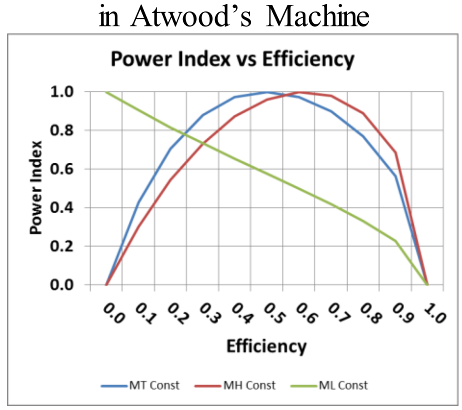
The circuit that demonstrates Jacobi's Law has similar limitations, and more so. In this case the "usable" energy is not, in fact, immediately stored for later use, but is, rather, immediately dissipated on some useful purpose such as running a motor. Now, it is easy to imagine that such a motor might ultimately raise a mass or otherwise take action to store energy reversibly, but it might not.

So, while neither the AM nor Jacobi's Law can be said to truly and fully demonstrate the maximum power principle as Odum came to understand it, they do clearly demonstrate the phenomenon of maximum rate of energy transfer at an intermediate efficiency. Such a phenomenon in operation as a part of every type of persistent energy transfer is a necessary precondition for the operation of the MPP. We will return to this issue later in section V.

Problems In Odum and Pinkerton's Paper of 1955

There is one error in the original paper by Odum and Pinkerton that must be described, discussed, and acknowledged, in order to be able to move on and avoid future arguments over unimportant issues, as has happened in the past. While we readily acknowledge such this error, we do not believe it mars the validity of the arguments in support of the MPP, only the presentation. In particular, there is one error in their presentation and use of the AM as an example of a strictly concave power-efficiency relationship, and that error caused some confusion and controversy leading to Silvert's paper of 1982, and Odum's rebuttal of 1983.

Figure 05 – Power Index vs Efficiency in Atwood's Machine



With respect to the AM, we can express both the average power and the efficiency as functions of M_L, M_H and M_T. Thus we have five variables, or five degrees of freedom. But to convert the relations between these variables to a 2-dimensional graph, we need to, somehow, eliminate three degrees of freedom – i.e. eliminate three variables. The definition of efficiency (see equation 4) provides one constraint, removing one degree of freedom, and the need for one of the variables. The definition of M_T as the sum of M_L and M_H (see equation 6) provides another constraint, removing one more degree of freedom, and the need for one more variable. The final constraint, in the example of the AM, is somewhat arbitrary. We may arbitrarily hold one of M_L, M_H or M_T constant, and in each case we constrain the manner in which power varies with efficiency in a different way. It happens that we get three different expressions for power as a function of efficiency depending on which of the three variables is held constant, as shown in equations 14 through 16 and in Figure 5.

$$\overline{P_U} = \sqrt[2]{\frac{Dg^3(M_T)^2}{2}} \times \sqrt[2]{\frac{\eta^2(1-\eta)}{(1+\eta)^3}} \;\; ; \; M_T\ constant \qquad \text{Equ 14}$$

$$\overline{P_U} = \sqrt[2]{\frac{Dg^3{M_H}^2}{2}} \times \sqrt[2]{\frac{\eta^2(1-\eta)}{(1+\eta)}} \;\; ; \; M_H\ constant \qquad \text{Equ 15}$$

$$\overline{P_U} = \sqrt[2]{\frac{Dg^3{M_L}^2}{2}} \times \sqrt[2]{\frac{(1-\eta)}{(1+\eta)}} \;\; ; \; M_L\ constant \qquad \text{Equ 16}$$

As can be seen from Figure 4, two of these result in a strictly concave curve, a map on the interval [0, 1], consistent with the prerequisites of hypothesis MPP II. It appears, in hindsight, that Odum and Pinkerton were discussing the curve for constant M_T, which peaks at a value of $\eta = 0.5$, whereas Silvert was only aware of the curve for constant M_H, which peaks at $\eta = 0.618$. Odum and Pinkerton incorrectly stated that the case of constant potential input (i.e. the case of constant M_H) was characterized by an efficiency of $\eta = 0.5$. This was possibly the result of a typographic error. They then went on to develop their argument for a specific class of linkage event types, in which they showed that the efficiency of that class of linkage event must always be less than or equal to $\eta = 0.5$. Since their identified example (constant M_H) was clearly in breach of their conclusions, much doubt was cast upon their arguments.

Remarkably, according to Calvert, a very similar confusion over the interpretation of Jacobi's Law was directly responsible for the historical delay of the invention of the dynamo by several decades. [13]

There is an important lesson which can be drawn from these unfortunate cases of confusion. The circumstances and the nature of the constraints upon the variable operation of the linkage event can determine the shape of the associated power-efficiency function. A corollary to Odum's proposition about the existence of such curves might be that any constraint on the variability of a type of linkage event that does not lead to a strictly concave map of power versus efficiency on the interval [0, 1] cannot lead to participation in a persistent pathway.

SECTION IV – Autocatalytic System Concepts and Terminology

To understand the MPP we need to think of an autocatalytic system from several perspectives. The following concepts are developed largely with an ecosystem in mind. But other kinds of natural autocatalytic systems such as metabolic systems or economic systems can be characterized by similar concepts, as can some logical autocatalytic systems such as computer models. In addition, many aspects of persistent phenomena which are not clearly autocatalytic can also be described using these ideas. With such intended generality of application in mind, some of the systems-oriented concepts and terminology remains intentionally somewhat loosely defined, as follows:

- **Persistence** – Many dynamic processes are of brief duration, run their course, and cease to function. A rock falling from a cliff face is such a dynamic process, and is not persistent in function. Some processes continue as long as there is a supply of inputs. Water in a stream falling from a cliff face is such a dynamic process, and is considered persistent, as long as there is a supply of water following the course of the stream. But, no process endures or persists forever. A dynamic process might be called persistent if there is a steady supply of inputs for a period of time that is extended substantially longer than what might be considered one execution of the process. So, a waterfall might be considered persistent if the flow in the stream has a duration substantially longer than the time required for a small amount of water, say a bucket of water, to fall from top to bottom. In short, persistence is a characteristic determined by comparison of relevant durations, and not an absolute characteristic. A persistent process continues to function during many iterations of some sub-processes which it encompasses.
- **Autocatalysis** – An autocatalytic system is a collection of mass of many types together with a set of interacting persistent processes that must consume some of their own products to remain persistent. The word "autocatalytic" originally referred to those inorganic chemical processes for which one of the reactants is also a product of the process, and so the process is persistent in the presence of a constant supply of the other required reactants. This concept can be extended to apply more generally to those dynamic systems which produce substances, compounds, objects, organisms or artifacts that are necessary for the continued functioning of the system. For example, most organisms, if not all, require a steady supply of ATP molecules in order to continue to function, and these molecules are produced and consumed as a normal part of the metabolic functioning of the organism. Without putting too fine a point on the definition, we consider a system to be autocatalytic if any of the components of the system are necessarily used to produce replacement components of a similar use and function.
- **Adaptability** – An adaptable system is an autocatalytic system which can continue to persist in an environment of changing quantities and types of inputs. It can enable new or variant processes that incorporate the new types of inputs, or adjust to shortages of inputs. It can discard old or variant processes that become less persistent, due to lack of suitable inputs, including those inputs that require autocatalysis. Resilient in the face of change, such systems are able to increase their own complexity over time as they adapt to changing inputs, including their own changing autocatalyzed inputs. Such systems are now referred to as complex adaptive systems (CASs), but such terminology was not in use when A. J. Lotka and H. T. Odum were writing about systems.
- **Linkable energy stores** – Every physical system consists of and contains a wide variety of types of linkable energy stores. By the word "linkable" we mean that the stores may be able to physically couple in some temporary or permanent fashion, and energy can be transferred from one to the other during that linkage event. Think of an organism as such an energy store, and think of a species as a type, class or category of energy store. In the flow of energy, we might consider the donor store to be "upstream" in the flow of energy, and the receiving store to be "downstream" in the flow. Each energy store can link to one or several other upstream energy stores from which it might receive a transfer of high-grade energy. It can also link to one or several other downstream energy stores to which endowments of high-grade energy might be transferred.

- **Linkage events, energy transfers and energy transformations** – A linkage event is a process by which two energy stores are coupled with one another, and energy is transferred from one energy store to another. In an ecosystem, the processes of reproduction (e.g. via egg laying, birth, or seed production) and consumption (e.g. via predation, parasitism, or decomposition) are linkage events by which energy is transferred from organism to organism. In the process of linking to a downstream store and transferring an amount of high-grade energy E_T, a portion E_W of the transferred energy is irreversibly transformed in type and degraded in quality in accordance with the second law of thermodynamics, and exhausted into the environment as waste heat. At the same time, the residual energy E_U is added to any previously accumulated store of still useful high-grade energy located in the downstream store. Here, $E_T = E_U + E_W$. The freshly (re-)energized downstream store is now able to use that energy for its own maintenance, or pass on some or all of this high-grade energy to the next downstream store in the chain, when a following downstream linkage is made. Each store contains an accumulation of some fraction of the flow of energy transferred to it from upstream store(s), and transfers some fraction of that on to downstream store(s) to which it may link. Denote an upstream energy store by α and a downstream energy store by β. There are three universal concepts that can be measured for every linkage event:
  - **Transformation Time** – All energy transformations require time, and no energy can ever be transformed or transferred instantaneously. It is difficult to say whether energy transformation of some kind is a necessary artifact of dynamic change, or whether it is in fact merely the tautological essence of dynamic change, reduced to a phrase. But, in either case, dynamic changes must take time, and so must energy transformations. The speed at which the transformation takes place will be determined by the nature of the coupling, the nature of the linkage event, between the two involved energy stores. Denote the transformation time of the linkage event between stores α and β as $T^{\alpha\rightarrow\beta}$. Note that this represents a duration $[0, T^{\alpha\rightarrow\beta}]$.
  - **Useful Power of a Linkage Event** – The average power, computed as the amount of high-grade energy that is transferred to the downstream energy store divided by the transformation time, is here referred to as the "useful power" of the linkage event. Denote the amount of still useful energy transferred in a linkage event as $E_U^{\alpha\rightarrow\beta}$. Then the useful power is calculated as $\overline{P(\eta)}^{\alpha\rightarrow\beta} \equiv E_U^{\alpha\rightarrow\beta}/T^{\alpha\rightarrow\beta}$.
  - **Transfer efficiencies** – Each transfer of energy from one store to another store may be characterized by an efficiency rating calculated as a ratio $\eta^{\alpha\rightarrow\beta} \equiv E_U^{\alpha\rightarrow\beta}/\left(E_U^{\alpha\rightarrow\beta} + E_W^{\alpha\rightarrow\beta}\right)$, where $E_W^{\alpha\rightarrow\beta}$ is the intrinsic second-law tax associated with the linkage event.
- **Types or Classes of Stores and Linkage Events** – Let us assume that we can classify all energy stores according to size, structure and content. Let us assume further that we can classify all linkage events according to their three universal characteristics. The goal of such a classification system is that all stores exhibiting the same universal characteristics in all linkage events belong to the same type of store, one store being replaceable by another store of the same type in any such linkage event with little or no difference in outcome. We can then extend the above concepts to types of stores and types of linkage events. Let A and B represent two types of energy stores. Let N be the number of linkage events between

elements of A and B, and let the linkage events be enumerated by the index *i*, so the stores involved in linkage event i are $\alpha_i \in A$ and $\beta_i \in B$. Then:

- **Expected transformation time** – The expected transformation time for the transformation from type A to type B can be defined as the average transformation time across all sampled transformations of that type, and would be calculated as $T^{A\rightarrow B} \equiv \left(\sum_{i=1}^{N} T^{\alpha_i \rightarrow \beta_i}\right)/N$. If the storage types A and B are properly defined, then $T^{A\rightarrow B}$ for the sample should be very close to the individual $T^{\alpha_i \rightarrow \beta_i}$ for each transformation *i*.
- **Expected Useful Power** – Similarly, the useful power associated with a linkage event between energy stores of types A and B would be the average across all sampled linkage events calculated as $\overline{P(\eta)}^{A\rightarrow B} \equiv \left(\sum_{i=1}^{N} \overline{P(\eta)}^{\alpha_i \rightarrow \beta_i}\right)/N$.
- **Expected Transfer Efficiency** – Finally, the efficiency of N such linkage events would be $\eta^{A\rightarrow B} \equiv \left(\sum_{i=1}^{N} \eta^{\alpha_i \rightarrow \beta_i}\right)/N$. By definition, these efficiencies are within the interval [0, 1]. For example, when a type of frog eats a type of mayfly one might find that the expected transfer efficiency of this type of linkage event is $\eta^{A(fly)\rightarrow B(frog)} = 0.35$. Here, the type of fly may be determined by size, by structure (i.e. species), and by content (chemical potential energy).

- **Pathways** – The system, as a whole, can be viewed as the composition of many branching and merging pathways through which the energy flows as it is initially captured from sunlight via photosynthesis, and subsequently stored, transported, accumulated, transformed, transferred and degraded by the system. In this perspective, a pathway consists of a list of linkable energy stores (objects) and linkage events (energy transformation events in which energy is both transferred, in part, and degraded, in part). Note that such a pathway may be purely conceptual, as the elements of the pathway may be destroyed (e.g. by predation) as the energy passes down through the pathway. So, the concept of pathway exists at two distinctly different logical levels:
  - **Realized pathway** – a list of stores (e.g. organisms) and linkage events (e.g. reproduction and predation events); and
  - **Idealized pathway** – a list of types of stores (e.g. species) and types of linkage events (e.g. predator-prey pairings).
- **The environment as energy source and energy sink** – The entire system of energy pathways exists in an environment in which there is a continually refreshed source of high-grade energy, such as sunlight, and the system has some ability to capture or absorb some portion of this energy, store it, transport, accumulate, transform and transfer it from store to store, and degrade it via its internal processes, before exhausting the degraded energy back into the environment as waste heat. The realized pathways within the system consist of transitory linkages between transitory energy stores. The idealized pathways within the system might be viewed as chains or webs of persistent types of linkages between persistent types of stores.
- **Persistence of pathways** – Stores, pathways, or the system as a whole may be destroyed or sustained during any energy transformation event or process. Those idealized pathways for which the destroyed stores are replenished from time to time will have a limiting rate of transfer and consumption of energy that is partially dependent on the mean rates of consumption and replacement or replenishment of the required stores, of each type, and the mean times required for energy transfers to be completed between stores. Those idealized pathways that are not sustained due to the destruction of stores in its chains without

replacement or replenishment – those idealized pathways will be transient. In those environments in which an abundance of stores is produced and continuously available, such as in the cytoplasm of a cell, or in an ecosystem – in those environments such idealized energy pathways will be relatively unconstrained, and energy will transfer along the pathway from store to store at a relatively high rate. We can extend the characteristic of persistence from processes (e.g. idealized pathways) to types of linkable stores (e.g. species), to types of linkage events (e.g. predation or gestations events), and to the system itself. Such a system that is persistent in all of these ways might itself be called persistent. An energy store is persistent if it survives most linkage events in which it is involved, or if the system quickly replaces it after it is consumed. A linkage event is persistent if the participating energy stores are persistent. The persistence of a pathway has already been defined, but can be restated here in different terms. An idealized pathway is persistent if some of the stores and linkage events of which it is constituted are persistent for a duration of time exceeding a few executions of the pathway. A system, then, is persistent if some of the idealized pathways of which it is constituted are persistent.

- **Differences of scale** – Each type of energy store may, in fact, be a type of system in its own right, consisting of persistent idealized pathways of linkable stores of a different kind at a smaller scale. For example, consider an organism that participates in the trophic web of an ecosystem and, at the same time, comprises many metabolic pathways determined by genetic heritage and, perhaps, circumstance of birth. Energy and matter pulse and flow along these metabolic pathways, lingering at some steps for mere moments, and at others for days, or longer, possibly for the lifetime of the organism. But all of this internal activity is entirely within a single step along a realized pathway in the trophic web. So we see that these concepts can apply simultaneously to overlapping and inter-tangled systems for which the characteristics vary greatly in function and in scales of time and space. Such is the nature of the biosphere in which we have evolved and in which we now live out our lives. If this biosphere is to be persistent, it must be continually renewed, highly adaptive, and responsive not just to environmental changes, but also to its own evolving internal structure of pathways of all types at all scales.
- **Ubiquity -** H. T. Odum's proposal implies that the maximum power principle (MPP) is operative in every persistent system at every type of persistent linkage event, and along every such persistent idealized pathway, at every scale, enabling the persistence and increase of power of each and every such pathway. Those idealized pathways characterized by more effective types of linkable stores and types of linkage events will increase their power, and their associated mass, eventually dominating the less effective pathways, capturing a greater share of the flow of mass and of energy.

Mixing of Metaphors

We are using a variety of metaphors, drawing from daily experiences, to describe concepts that are just beyond our experience. These metaphors include stores, paths, chains and webs. But each of these metaphors fails, in some way, to fully capture the entirety of the concept it represents. E.g. we are using the metaphor of a "pathway" to describe something that is only distantly reminiscent of a true pathway, as one might find through a woodland. We are also using the metaphor of a spider's "web" to describe multiply-intersecting pathways, though a spider would certainly not view its web in that fashion. Not to be too confusing, we also need to explore briefly, the other metaphor commonly used in ecological discussions, and that is of a chain.

Consider a linked chain of energy stores, each link, or store, receiving energy from the store ahead of it via a linkage event, and passing a fraction of that energy on to the next store that follows after in another linkage event. In the metaphor of a chain, stores are replaced by chain links. The analogy to a chain is appropriate in one way in particular, as you can imagine that, like links in a chain, each store is somehow intimate with the store immediately preceding it, from which it receives energy, and then, is somehow intimate with the store immediately following it, to which it passes on a portion of that energy. The nature of the linkage event may be through gestation, through consumption, predation or parasitism, or merely through molecular reconfiguration. The connection between linkable stores need not be permanent, and, in fact often is not, but may exist only briefly. As the two links randomly connect, energy is transferred, and the connection is then broken, never to be repeated. The links may be organic molecules in a metabolic pathway, or organisms in a tropic web. In such cases, the energy stores that actually form the links in the chain may each be individually formed in one energy transfer, survive several transfers, but then destroyed in another, so the store-to-store linkage events may form temporarily, connect briefly, transfer energy, and disassemble in the process. For example, a frog (a store, or link) may eat (linkage event) many flies (stores) before laying (linkage event) eggs (stores), after which it is ultimately eaten (linkage event) by a heron (store), which is then eaten (linkage event) by a fox (store). The links in such chains are forged by reproduction, and grown in size by consumption and accumulation, and destroyed by death. So the downstream recipients of energy from an organism would include offspring, predators, parasites, and decomposing agents of various sorts.

Thus, we find ourselves using at least three different useful but not entirely adequate metaphors to capture these somewhat elusive concepts. In the chain metaphor, we talk about chains, links, and linkable events – a metaphor that is able to capture the intimacy of the connections between persistent types of linkable objects that exist in great variety in the system. In another metaphor we talk about pathways, and possibly steps along the path, that might shift and move over time through a forest with many possible paths – a metaphor that implies the time-regulated traversal of the path with many optional branches along its course. In yet another metaphor we talk about a web of many paths that branch and rejoin in almost infinite variety.

Lotka argued that such system-wide persistent pathways will evolve, under the action of a Darwinian kind of natural selection, such that they process more and more energy at ever higher rates, until they achieve a maximum power determined by limits on the energy source, or limits on the appropriate kinds of matter needed to capture, hold, and transfer the energy. For example, if a species of frog evolves so as to garner a greater share of the available stream of energy than before, it will flourish, while some other species (of frog?) will have less energy and encompass less mass and their numbers will wane. And so, at each link in the chain, those species that outcompete the others will expand the flow in their portion of a pathway or create alternate pathways that degrade energy at higher rates. As the species of frogs, their prey, and their predators evolve, the associated linkage events and pathways also co-evolve, as does the system as a whole, to maximize the transfer and consumption of energy. Should there be some as-yet untapped persistent source of usable energy in the environment, then, when some species adapts to access that energy, the system as a whole will also adapt to harvest and process that energy source.

Lotka further argued that a serious constraint on the ability to capture and process energy is the availability of appropriate matter. For example, photosynthesis cannot happen without pathways providing access to the proper chemical elements from which leaves and chlorophyll may be formed. But, the discovery and developing availability of new sources or forms of matter that may be exploited enables the capture and processing of more energy as plants adapt, which again will cause some system-wide pathways to grow, exploiting that matter to the fullest extent, and exploiting that energy to the fullest extent. This is evident in simple fashion when one puts fertilizer on a garden plot. It is evident in more complex fashion when one contemplates the emergence of life from the sea to cover the previously bare and lifeless continents. Thus, the system will evolve to capture and degrade energy at a high rate, and to cycle and recycle matter of many sorts at higher rates.

The Extension to Economies

Odum also argued [9, 12] that all of these concepts were equally applicable to social systems and social institutions of all kinds, including that social institution we call our economy.

Beyond energy and mass, there are others types of resources available to a social system that enable the increased flow of energy and mass through it, and we call these resources "human capital". In this group are included human talents, skills, and education. We argue that a shortage of such human capital places a constraint on the flow of energy. If some untapped source of human capital is discovered (e.g. a scientific principle), then the system will adjust to develop that capital (elaboration and transfer of skills and knowledge via educational institutions), exploiting the human capital to the fullest extent, and so increasing the flow of mass and energy through the society.

This leads into one final perspective we can take on these matters. Masses of various kinds (molecules, organisms, commercial artifacts) often form the medium or vector by which energy is transported and stored. The means of interaction of these masses determines the dynamic characteristics of the linkage events in which they are involved. Esoteric forms of capital of various kinds (human capital, intellectual property, cash) often alter the efficiency and effectiveness of linkage events within human economies. The facilitating effects of this capital alters the dynamic characteristics of the linkage events in which it is applied. It appears that energy flows are the key concept, matter flows provide a vector, and capital flows facilitate both matter and energy flows. So, we see these autocatalytic systems operating at three somewhat overlapping and not very distinct levels: energy stores and flow; matter stores and flows; and capital stores and flows. With some care, perhaps one could augment most of the above concepts around persistent energy stores and flows with more detailed considerations of the roles of matter and capital.

This would mean that the MPP is the engine of growth, not just at the cellular level, but at the level of national and global economies. Odum argued [12] that the free-market concept has harnessed the power of the MPP, and that is the source of the increasing rate of flow of mass and energy through our global economies. We would also argue, in line with Joseph Tainter's view of complexity in society's institutions [14], that the MPP is the engine that produces ever increasing complexity in our modern global societies.

But, Odum also argued that humanity has, perhaps unwittingly, aligned our world view, our social institutions, our policies, and our practices to enable and enhance the effects of the MPP. This would seem to be a very pertinent observation, given the current state of world affairs.

SECTION V – Restatement of the Maximum Power Principle

With these ideas in mind, then, we are now in a position to restate the maximum power principle (the MPP) as we believe Odum envisioned it. To do this we recast the concepts found in the writings of Lotka and Odum, presenting them as falsifiable hypotheses. The wording here is specifically applicable to adaptive autocatalytic physical systems, but a variation may also be applicable to simple non-adaptive but persistent physical systems, or to purely logical autocatalytic systems, such as autocatalytic computer models. The role of human capital in economic systems is not part of this restatement, but would require additional consideration and possibly some conceptual changes.

HYPOTHESIS MPP I – APPLICABILITY

All persistent adaptive autocatalytic physical systems are characterized by energy fluxes through persistent types of energy stores, and persistent energy pathways through which energy flows from storage type to storage type.

H.T. Odum believed that the MPP applied to every sort of persistent physical system, from purely physical systems (such as star systems, galaxies, hurricanes and tornadoes) to biological systems (such as metabolic systems, organisms and ecosystems) to sociological systems (such as social organizations and practices, and economies). We herein exclude the non-adaptive persistent physical systems even though Odum provided some examples of such, since the MPP may apply to some such systems, but, possibly, not all.

HYPOTHESIS MPP II – LINKAGE EVENTS

Within all persistent adaptive autocatalytic physical systems, all classes of persistent linkage events within persistent energy pathways are characterized by strictly concave power-efficiency functions for which power is maximized at some intermediate efficiency.

Odum published his version of the strictly concave curve associated with the AM as an example of such a curve in his paper of 1955, as discussed in section III. (See figures 1 and 4.)

There are two aspects to this hypothesis that bear discussion:

- The shape of the power-efficiency curve depends entirely on the nature of the constraints that are placed on the potential variations in the linkage between the two stores. To produce a strictly concave power-efficiency curve at the right-hand side of the interval [0, 1], there must be some sort of direct relationship between expected transfer time and expected efficiency – more efficiency must require a longer duration of time. This is not an unusual relationship. Within human experience it is quite commonly understood that a any job that takes time is done less well in a short time period, and done better in a longer time period. For example, suppose you are shoveling sand out of a sand box, and suppose you define the efficiency as the quantity removed over the total quantity. Then you might expect an removal efficiency of 50% in an hour, but an efficiency of 100% might require days of meticulous removal of grains of sand, and associated dust. Of course, power will decline dramatically as efficiency improves, since there is a limit on the volume of sand to be removed, but there is not limit on the time to complete the task. However, if such a relationship between time and efficiency does not exist, then the concave curve is not possible. Of course, other sorts of power-efficiency curves might exist, as shown in section III with respect to the AM. Such curves may not be strictly concave.

- If there is a type of linkage event, and an associated natural constraint on its variability, such that the power-efficiency curve is not strictly concave, then we offer this proposition that the linkage event cannot be a persistent element of a persistent pathway. Our evidence for this rests only on two observations. In the two computer models examined so far, the persistent linkage events exhibited a concave power-efficiency function.

HYPOTHESIS MPP III – EVOLUTIONARY TENDENCIES

All persistent adaptive autocatalytic physical systems evolve to capture and degrade energy at a maximal possible rate consistent with available inputs. In contrast, the efficiency of the linkage events will not be maximized but will tend towards some intermediate value.

In contrast to Boltzmann's expressed opinions about maximized efficiency of species, neither Lotka nor Odum believed that efficiency is maximized in the system as a whole. Rather, they argued that energy throughput was maximized in the system. The question as to when efficiency is maximized, and when power is maximized, is a difficult one to unravel.

We know that efficiency is maximized in many aspects of any system's operation. For example, most biological functions of most species of organisms are highly efficient in their function. The human body is remarkably efficient at walking or running. Our lungs are efficient air exchangers. Our red blood cells are efficient at absorbing, carrying and releasing oxygen. In many of these aspects, efficiency seems to be the primary characteristic of adaptive evolutionary advantages. Each of these sorts of efficiencies will result in more benefit at a cost of less energy consumption. We can say that when it comes to metabolic expenses, our bodies are very efficient.

But, we also know that in one regard, nature is very profligate. If we assume that nature is in a stationary state, with the population of all organisms at carrying capacity, then each organism should only have need to produce one offspring as a replacement. However, the nature of evolution, and the struggle for survival, demands that all organisms compete for a place in the next generation, and so we produce far more offspring than have any reasonable chance for survival, with the apparent affect that most offspring will not survive to reproduce. A single mushroom will produce millions of spores. A pine tree will live anywhere from a hundred years to several thousand years, producing hundreds or thousands of seeds every year. Oceanic biota such as fish, coral polyps, or krill produce immense numbers of offspring filling the oceans with their tiny bodies. In every case, the parent organism has accumulated and stored energy in the germ of life that is to become the replacement organism. Ultimately, on average, all but one will succumb to predation, parasitism or hunger prior to becoming reproductive, and all of that accumulated energy will move down the pathways of the trophic web to the next trophic level. For that one offspring that survives, it must fight for its fair share of the flow of energy, accumulate what energy it can, and in turn produce its own energized offspring. Those organisms that do not maximize their reproductive chances are outcompeted, and disappear from the biosphere.

So, we have these two extreme characteristics of the products of biological evolution: metabolic processes associated with growth and maintenance are adapted to be most efficient, while reproduction is profligate in its apparently excessive energy costs. The amount of "still useful" energy passed on to offspring is maximized, in agreement with the maximum power principle.

For this hypothesis to be true, we must consider two associated and dependent inferences.

HYPOTHESIS MPP III-A – CO-EVOLUTION OF COMPONENTS

Persistent energy pathways, together with the types of energy stores and of linkage events that they comprise, must all co-evolve to achieve overall maximum power at the level of the entire system.

The evolution of species is well understood. One may ask how a linkage event, a type of energy transfer event, can evolve. Consider the relationship in the trophic web between one species of fly and one species of frog that predates on the fly. When a frog eats a fly, the event has a duration over which the fly is digested, and an associated efficiency of transfer of energy from the fly to the frog as "still useful" energy. One could then plot the power of the event versus the efficiency of the event. Suppose one member of the population of frogs enjoys an adaptive genetic mutation such that it garners more "still useful" energy from each fly it eats, thereby increasing the number and viability of the eggs it lays, and increasing the number of its offspring that survive. This altered efficiency, whether upwards or downwards, has increased the power of this type of linkage event. The new population of frogs will likely outcompete the population out of which it arose, and a new pathway will outcompete the old pathway. So we see that the process of evolution of species also implies that the energy transfer mechanisms between species (the types of linkage events) and the pathways that are composed of stores and linkage events will all benefit from a positive adaptation.

HYPOTHESIS MPP III-b – EXPANDING SCOPE AND COMPLEXITY

Any such system for which its operation is constrained by a shortage of suitable inputs, but for which alternative inputs are accessible, will tend to adapt its pathways so as to access those inputs, with the effect that energy throughput will increase.

This hypothesis is best understood in consideration of a number of examples. Consider an ecosystem which is already functioning at maximum power consistent with its current composition and operation. Suppose there are untapped sources of energy, such as non-decomposable detritus. Over time, the organisms that are able to decompose that detritus will tend to develop, thereby increasing the rate of degradation of the energy flowing through the ecosystem. By this means the energy pathways adapt to expand the scope of operation of the ecosystem, and thereby increase the power of the ecosystem. Or suppose there are untapped sources of sunlight falling on rocky soil or on tree trunks. Over time, organisms will tend to develop that are able to live in those easily accessible microclimes, capture the energy in that sunlight, and so increase the power of the system.

But the ecosystem may be constrained in other ways. The persistent energy pathways consist of stores that may be consumed regularly, as in a trophic web, and those stores must be replaced regularly using available mass. The total biomass is then a constraint on the size of the ecosystem, and a constraint on its power. If there are suitable deposits of mass that can be exploited, the pathways of the ecosystem will tend to adapt to exploit that mass, increasing the total biomass, and thereby increasing the power of the ecosystem.

Finally, in the case of an economic system, a similar process of adaptation of pathways and increase in scope of operation occurs, but with respect to untapped "human capital". Let us indulge in a thought experiment. Suppose an economy is functioning at maximum power, and all sources of energy and mass are being tapped at full capacity. The economy is in some kind of steady state. Then, suppose that there is some undeveloped human capital – some undeveloped skill, or knowledge, or technology – that could enable the economy to access as-yet-untapped sources of energy or mass, and thereby increase the size and scope of the economy, and increase its power. Under such conditions, the economy, in the persons of researchers and engineers, would develop the needed technology, and thereby adapt its pathways such that the scope and power of the economy would increase.

In all of these examples in which we consider the effects of accessible but untapped sources of energy, of mass, or of human capital, the pathways of the system will adapt to increase the scope, complexity, and power of the system. By increased scope we mean an increase in the total volume of energy, of mass, and of capital exploited. By increased complexity we mean an increase in the numbers of types of stores, of types of linkage events, of alternative pathways, and the lengths of those pathways. By increased power, we mean an increase in the rate of degradation of the energy flowing through the system.

Since, as a general principle, the rate of rise of entropy is proportional to the rate at which energy is degraded, such a system would not just be functioning at maximum power, but also at a maximum rate of energy production. Others have proposed that the Maximum Entropy Production Principle (or MEPP) be considered as a fourth law of thermodynamics, in place of Odum's MPP. We see, by this argument, that these two concepts may be two sides of the same coin.

SECTION VI – Problems and Implications

We believe that the above presentation reasonably captures H. T. Odum's vision of the nature and role of the MPP in persistent autocatalytic systems of all kinds. In our opinion the above hypotheses should be the subject of widespread research, and we expect that they will be validated, in the sense that they will survive all attempts to falsify them with very few additional qualifications placed on their applicability.

However, for these propositions to be viewed as falsifiable hypotheses, in accordance with the ideas of Karl Popper, some ambiguities and difficulties first need to be resolved. But, that may be difficult to do when making general statements applicable to a very wide range of complex systems.

- The first problem is conceptual: the best way to classify 'types of linkage events' when considering metabolic pathways involving organic and non-organic molecules may not be applicable to linkage events in trophic chains in ecosystems, or linkage events in supply and distribution chains in economic systems, so each and every system may require its own classification system for types of stores and linkage events.
- The second problem is one of practicality: when instantiations of linkage events are so transitory, when classification by event type is so problematic, and when pathways themselves are so ephemeral, how does one effectively collect data to support or refute the MPP?

Odum and Pinkerton addressed both of these problems, but only in part, by making certain scope-reducing assumptions and developing an argument that applied to a specific class of energy transfers, i.e. a specific class of linkage events, drawn from physical and biophysical systems. We will say more about that specific argument later.

Application to Ecosystems

In the context of ecosystems, natural selection, as described by Darwin, will play a role in determining the evolving power-efficiency characteristics of each type of linkage event in a trophic web such that energy accumulated for and spent on reproduction is maximized, and energy spent on other necessary metabolic processes is minimized. However, since most offspring are eaten by predators and parasites, this amounts to simultaneously maximizing the rate of flow of energy through the tropic web between species as well as within a species. Classically, an organism has a phenotype determined by its genotype, and Darwinian natural selection is effected upon the phenotype of each organism. The result of natural selection at this level is one of two outcomes: either the organism lives to pass its genes on to the next generation, and so contributing to an increase in the number of organisms with similar phenotype, or it dies, potentially contributing to the decline of numbers of organisms with similar phenotype. But it is more complicated than that. It is the evolutionary imperative of each organism to produce many more offspring than are needed to replace itself in order to maximize the probability that its genes will be propelled into the next generation. This is part of the phenotype on which natural selection takes action. Since most offspring are killed and eaten prior to reproduction, this reproductive strategy provides a wealth of energy for predators, parasites and decomposers.

At the lower scale of the metabolic pathways within a species – both those having a role in feeding or reproduction, and those with less relevant metabolic purposes – these pathways are part of the species' phenotype on which Darwinian natural selection operates. Each such metabolic pathway exists in the organism because it has performed an adaptive function for the species in the past, and that function has required the expenditure of energy in its performance. The pathway persists because it delivers the required energy to the required site in a timely fashion. If multiple pathways exist enabling the same phenotypic function, the most effective pathway confers the most advantage. These more effective pathways will be indirectly selected as an organism engages in the struggle for survival. Thus, each link in each metabolic pathway is also indirectly subject to Darwinian natural selection, and must transfer maximum useful power at minimum energy cost.

At the higher scale of pathways through the trophic web of the ecosystem, it is not, strictly speaking, Darwinian natural selection, but a variation on it, that is active. An instantiation of a pathway is a collection of organisms, and energy is passed along through the web via reproduction (within a species) or death (between species). So, when an organism is captured and eaten by a predator may be considered a negative selection for the prey species, it is a positive selection for the predator species. However, ultimately, it is a positive selection for this particular type of linkage event (this predator/prey pair), increasing the flow of energy along this portion of the pathway at the expense of some other portion of the pathway. This is not a linear phenomenon, as there is clearly feedback that affects the survival chances of both predator and prey species, as exemplified in the greatly over-simplified but instructive Lotka-Volterra predator-prey model.

So, how, one may ask, does natural selection work at this level – the level of pathways through tropic webs? For those linkage events for which the phenotypes of both predator and prey species lead to positive selection due to classical Darwinian natural selection, for those linkage events, the flow of energy through that portion of the pathway will grow in proportion to, and at the expense of, the energy flow through competing pathways, and all downstream organisms will have greater opportunity to propel their genes into the future. Greater success for any linkage event leads to greater opportunity for success for all succeeding organisms in the chain. Thus, a Darwinian-like form of natural selection will confer advantage on those pathways that can garner and pass on the larger share of available energy flows.

SECTION VII – Summary

In this paper we have identified A. J. Lotka and H. T. Odum as the originators of the concept of the maximum power principle (MPP), which Odum proposed should be considered the fourth law of thermodynamics. We have examined the two physical examples of Atwood's Machine and Jacobi's Law in which power-efficiency functions exhibit a strictly concave form, as discussed by Odum. We have identified a range of concepts and terminology that are needed to discuss the MPP. We have restated the MPP as a series of three falsifiable hypotheses describing the behaviour of persistent autocatalytic systems. We have examined some of the issues around testing and attempting to falsify these hypotheses.

We believe that the MPP is not only a fitting candidate to be considered as a fourth law of thermodynamics, but that it has much wider applicability outside of that field of interest, as a fundamental explanatory tool for understanding the dynamics of developing national and global economies.

SECTION VIII – References

[1] Lotka, A. J., "Contribution to the energetics of evolution", Biology, Vol. 8, pp 147-151, 1922.

[2] Odum, H. T., Pinkerton, R. C., "Time's speed regulator: The optimum efficiency for maximum power output in physical and biological systems", American Scientist, Vol. 43, No. 2 (APRIL 1955), pp. 331-343.

[3] Holmes, S. J., "The principle of stability as a cause for evolution." The Quarterly Review of Biology, Vol. 23, No. 4 (Dec., 1948), pp. 324-332.

[4] Onsager, L., "Reciprocal Relations in Irreversible Processes. I.", Phys. Rev. 37, 405–426 (1931).

[5] Onsager, L., "Reciprocal Relations in Irreversible Processes. II.", Phys. Rev. 38, 2265–2279 (1931).

[6] Tribus, M., "Thermostatics and Thermodynamics: An Introduction to Energy, Information and States of Matter, with Engineering Application.", D. Van Nostrand and Company, Inc., Princeton, NJ, p. 641ff (1961).

[7] Smith, C. C., "When and how much to reproduce: The trade-off between power and efficiency.", Amer. Zool., 16: 763-774 (1976).

[8] Silvert, W., "The theory of power and efficiency in ecology.", Ecol. Modelling, 15:159-164 (1982).

[9] Odum, H. T., "Maximum power and efficiency: A rebuttal", Ecol. Modelling, 20: 71-82 (1983).

[10] Odum, H. T., "Biological circuits and the marine systems of Texas. In Burgess and Olsen (Editors), Pollution and Marine ecology", John Wiley, NY, pp. 99-157 (1967).

[11] Odum, H. T., "Ecological and General Systems: An Introduction to Systems Ecology – Revised Edition", Colorado University Press, 644 pp. (1994).
[12] Odum, H. T., "Environment, Power and Society", Wiley-Interscience, NY, 336 pp. (1971).
[13] Calvert, J. B., "Jacobi's Theorem – Also known as the Maximum Power Transfer Theorem: Misunderstanding of it retarded development of dynamos", http://mysite.du.edu/~jcalvert/tech/jacobi.htm, Accessed 2015, (2001).
[14] Tainter, J. A., "The Collapse of Complex Societies. In the series NEW STUDIES IN ARCHAEOLOGY", Cambridge University Press, NY (1988).